KVPY QUESTION PAPER-2018 (STREAM SA)

Part – I

One - Mark Questions

Date : 04 / 11 / 2018

MATHEMATICS

1. The number of pairs (a, b) of positive real numbers satisfying $a^4 + b^4 < 1$ and $a^2 + b^2 > 1$ is

(A) 0 (B) 1 (C) 2 (D) more than 2

Ans. **[D]**

Sol. $a^4 + b^4 < 1$ ..... (i)

$a^2 + b^2 > 1$

$-a^2 - b^2 < -1$ ..... (ii)

(i) + (ii) $\Rightarrow$

$a^4 - a^2 + b^4 - b^2 < 0$

$(a^2 - \frac{1}{2})^2 + (b^2 - \frac{1}{2})^2 - \frac{1}{4} - \frac{1}{4} < 0$

(a^2, b^2) lies inside circle with centre $\frac{1}{2}, \frac{1}{2}$ & radius $\frac{1}{\sqrt{2}}$,

$\Rightarrow$ more than 2 solutions

2. The number of real roots of the polynomial equation $x^4 - x^2 + 2x - 1 = 0$ is

(A) 0 (B) 2 (C) 3 (D) 4

Ans. **[B]**

Sol. $x^4 = x^2 - 2x + 1$

$x^4 = (x - 1)^2$

$x^2 = x - 1$, $x^2 = 1 - x$

$x^2 - x + 1 = 0$, $x^2 + x - 1 = 0$

$D < 0$ $D > 0$

no roots 2 roots

3. Suppose the sum of the first m terms of arithmetic progression is n and the sum of its first n terms is m, where $m \neq n$. Then the sum of the first (m + n) terms of the arithmetic progression is

(A) 1 – mn (B) mn – 5 (C) –(m + n) (D) m + n

Ans. **[C]**

Sol. $\frac{m}{2}(2a + (m-1)d) = n$ ..... (1)

$\frac{n}{2}(2a + (n-1)d) = m$ ..... (2)

eq. (1) – eq. (2)

$\frac{1}{2}2a(m-n) + d(m^2 - n^2 - m + n) = n - m$

$\frac{1}{2}[2a + (m + n - 1)d] = -1$

$\frac{m+n}{2}[2a + (m + n - 1)d] = -(m + n)$

$S_{m+n} = -(m+n)$

4. Consider the following two statements :

I. Any pair of consistent linear equations in two variables must have a unique solution

II. There do not exist two consecutive integers, the sum of whose squares is 365. Then

(A) both I and II are true (B) both I and II are false

(C) I is true and II is false (D) I is false and II is true

Ans. **[B]**

Sol. I. Consistent linear equations do also have infinite solutions.

II. $k^2 + (k+1)^2 = 365$

$2k^2 + 2k + 1 = 365$

$2k^2 + 2k - 364 = 0$

$D = 4 - 4 \times 2(-364)$

$\sqrt{D} = 54$

$k = \frac{-2 \pm 54}{4}$

$k = \frac{-2-54}{4} = -14 \Rightarrow$ consecutive integer are –14, –13

$k = \frac{-2+54}{4} = 13 \Rightarrow$ consecutive integer are 14, 13

Hence statement II. Is false

5. The number of polynomials p(x) with integer coefficients such that the curve y = p(x) passes through (2,2) and (4, 5) is

(A) 0 (B) 1 (C) more than 1 but finite (D) infinite

Ans. **[A]**

Sol. let $y = a_n x^n + a_{n-1} x^{n-1} + \text{--------} + a_0$

$2 = a_n 2^n + a_{n-1} 2^{n-1} + \text{--------} + 2a_1 + a_0$ (1)

$5 = a_n 4^n + a_{n-1} . 4^{n-1} + \text{--------} + 4a_1 + a_0$ (2)

eq. (2) – eq. (1)

$3 = a_n(4^n - 2^n) + a_{n-1}(4^{n-1} - 2^{n-1}) + \text{--------} +$

$2a_1$ odd = even (Q all coefficient are integers)

not possible ⇒ no such polynomial will exists

6. The median of all 4-digit number that are divisible by 7 is

(A) 5497 (B) 5498.5 (C) 5499.5 (D) 5490

Ans. **[B]**

Sol. 1001, 1008 ---------- 9996

$9996 = 1001 + (n - 1) \times 7$

$n = 1286$

$$\text{median} = \frac{643^{th} \text{ term} + 644^{th} \text{ term}}{2}$$

$$= \frac{(1001 + 642 \times 7) + (1001 + 643 \times 7)}{2}$$

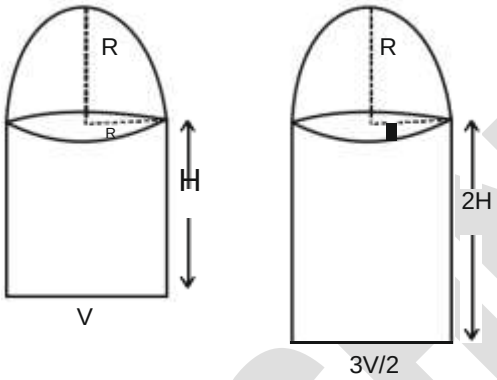
$$= \frac{2002 + 4494 + 4501}{2} = 5498.5$$

7. A solid hemisphere is attached to the top of a cylinder, having the same radius as that of the cylinder. If the height of the cylinder were doubled (keeping both radii fixed), the volume of the entire system would have increased by 50%. By what percentage would the volume have increased if the radii of the hemisphere and the cylinder were doubled (keeping the height fixed)?

(A) 300% (B) 400% (C) 500% (D) 600%

Ans. **[C]**

Sol.



$V = \pi R^2 H + \frac{2}{3}\pi R^3$ …….. (i)

$\frac{3V}{2} = \pi R^2 (2H) + \frac{2}{3}\pi R^3$ …….. (ii)

(ii) – (i)

$\Rightarrow \frac{V}{2} = \pi R^2 H$

$V` = \pi(2R)^2 H + \frac{2}{3}\pi (2R)^3$

$V` = 4\pi RH + \frac{2}{3}\pi R^3 \times 8$

$= 2V + 4V$

$= 6V$

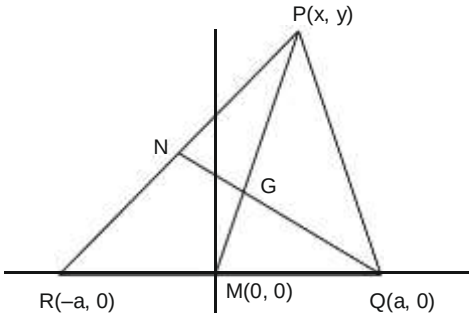
$$\% \text{ change} = \frac{6V - V}{V} \times 100 = 500\%$$

8. Consider a triangle PQR in which the relation $QR^2 + PR^2 = 5\,PQ^2$ hold. Let G be the point of intersection of medians PM and QN. Then $\angle$ QGM is always

(A) less than 45° (B) obtuse (C) a right angle (D) acute and larger than 45°

Ans. **[C]**

Sol.



$QR^2 + PR^2 = 5PQ^2$

$4a^2 + (x + a)^2 + y^2 = 5[(x - a)^2 + y^2]$

$\Rightarrow x^2 - 3ax = -1 - y^2$

$G = \left(\frac{x}{3}, \frac{y}{3}\right)$

$m_{GM} = \frac{\frac{y}{3}}{\frac{x}{3}} = \frac{y}{x}$

$m_{QG} = \frac{\frac{y}{3}}{\frac{x}{3} - a} = \frac{y}{x - 3a}$

$m_{QG} \cdot m_{GM} = \frac{y^2}{x^2 - 3ax} = -1$

$\angle QGM = 90^\circ$

9. Let a,b,c be the side-lengths of a triangle, and *l*, m, n be the lengths of its medians. Let $K = \frac{l + m + n}{a + b + c}$. Then, as a, b, c vary, K can assume every value in the interval

(A) $\left(\frac{1}{4}, \frac{2}{3}\right)$ (B) $\left(\frac{1}{2}, \frac{4}{5}\right)$ (C) $\left(\frac{3}{4}, 1\right)$ (D) $\left(\frac{4}{5}, \frac{5}{4}\right)$

Ans. **[C]**

Sol. $K = \frac{l + m + n}{a + b + c}$

For any triagle we know that

$\frac{3}{4}(a + b + c) < l + m + n < a + b + c$

$\frac{l + m + n}{a + b + c} \in \left(\frac{3}{4}, 1\right)$

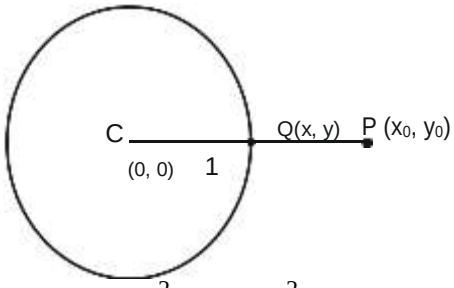
10. Let x_0, y_0 be fixed real numbers such that $x_0^2 + y_0^2 > 1$. If x,y are arbitrary real numbers such that $x^2 + y^2 \le 1$, then the minimum value of $(x - x_0)^2 + (y - y_0)^2$ is

(A) $\left(\sqrt{x_{02} + y_{02}} - 1\right)^2$ (B) $x_0^2 + y_0^2 - 1$ (C) $(|x_0| + |y_0| - 1)^2$ (D) $(|x_0| + |y_0|)^2 - 1$

Ans. **[A]**

Sol. (x_0, y_0) lies outside and

(x, y) lies inside or on the circle $x^2 + y^2 = 1$



$\min \{(x - x_0)^2 + (y - y_0)^2\}$

$= (CP - 1)^2$

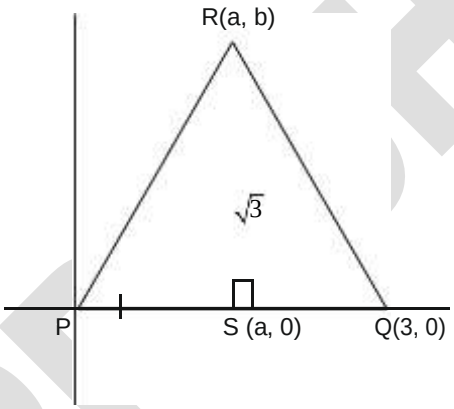
$= \left(\sqrt{x_{02} + y_{02}} - 1\right)^2$

11. Let PQR be a triangle in which PQ = 3. From the vertex R, draw the altitude RS to meet PQ at S. Assume that RS = $\sqrt{3}$ and PS = QR. Then PR equals

(A) $\sqrt{5}$ (B) $\sqrt{6}$ (C) $\sqrt{7}$ (D) $\sqrt{8}$

Ans. **[C]**

Sol.



$b = \sqrt{3}$

PS=QR

$a^2 = (a - 3)^2 + 3$

$a = 2$

$PR = \sqrt{a^2 + b^2}$

$= \sqrt{4 + 3}$

$= \sqrt{7}$

12. A 100 mark examination was administered to a class of 50 students. Despite only integer marks being given, the average score of the class was 47.5. Then, the maximum number of students who would get marks more than the class average is

(A) 25 (B) 35 (C) 45 (D) 49

Ans. **[D]**

Sol. Total marks of students = 50×47.5

$= 2375$

If 48 marks are obtained by students then max. No. of students can be = $\frac{2375}{48} = 49.479$

Means no. of students are 49.

13. Let s be the sum of the digits of the number $15^2 \times 5^{18}$ in base 10. Then

(A) $s < 6$ (B) $6 \leq s < 140$ (C) $140 \leq s < 148$ (D) $s \geq 148$

Ans. **[B]**

Sol. $N = 15^2 \times 5^{18} = 3^2 \times 5^{20} = \frac{(10-1).2^{20} \times 5^{20}}{2^{20}}$

$N = \frac{(10-1).10^{20}}{1024 \times 1024} = \frac{10^{21} - 10^{20}}{1024 \times 1024}$

Number of digits in N is < 16

It means no. of digits in N is 15

If all digit will be 9 then also max. sum is 135.

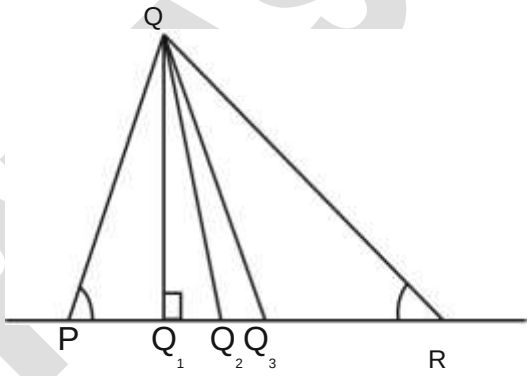
$\Rightarrow 6 \leq s < 140$

14. Let PQR be an acute-angled triangle in which PQ < QR. From the vertex Q draw the altitude $Q Q_1$, the angle bisector $Q Q_2$ and the medium $Q Q_3$ with Q_1, Q_2, Q_3 lying on PR. Then

(A) $PQ_1 < PQ_2 < PQ_3$ (B) $PQ_2 < PQ_1 < PQ_3$ (C) $PQ_1 < PQ_3 < PQ_2$ (D) $PQ_3 < PQ_1 < PQ_2$

Ans. **[A]**

Sol.



15. All the vertices of rectangle are of the form (a, b) with a, b integers satisfying the equation $(a-8)^2 - (b-7)^2 = 5$. Then the perimeter of the rectangle is

(A) 20 (B) 22 (C) 24 (D) 26

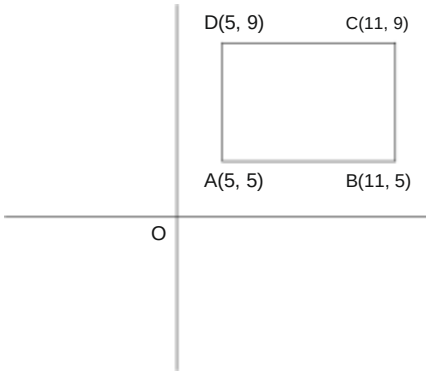
Ans. **[A]**

Sol. $(a-8)^2-(b-7)^2=5$

$A^2-B^2=5$ $(A=(a-8), B=(b-7))$

$A=\pm 3, B=\pm 2$

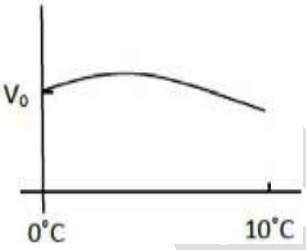
$\Rightarrow a=11, 5 ; b=9, 5$

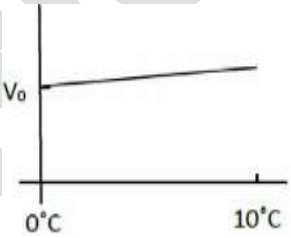


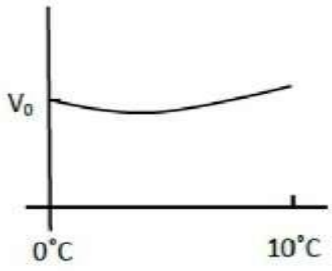
$\Rightarrow$ perimeter $=2\times(6+4)=20$

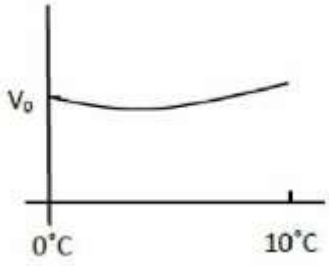
PHYSICS

16. A block of wood is floating on water at 0°C with volume V_0 above water. When the temperature of water increases from 0 to 10 °C, the change in the volume of the block that is above water is best described schematically by the graph

(A) 

(B) 

(C) 

(D) 

Ans. **[A]**

Sol. Change in volume of block above water level change can be due to following reason :-

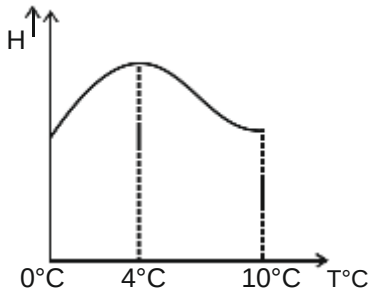
1) The density/volume of fluid changes due to increase or decrease in temperature

2) The volume of material which is immersed changes due to temperature change

According to Archimedes principal upward Bouyant force acting on block is equal to weight of fluid displaced.

When density increases, upward force increases and height of block above water increases and vice versa. For water

0ºC – 4ºC	density ↑	Height above water ↑
4ºC – 10ºC	density ↓	Height above water ↓



17. A very large block of ice of the size of a volleyball court and of uniform thickness of 8m is floating on water. A person standing near its edge wishes to fetch a bucketful of water using a rope. The smallest length of rope required for this is about

(A) 3.6 m (B) 1.8 m (C) 0.9 m (D) 0.4 m

Ans. **[C]**

Sol. If ice float on water then

$W = F_b$

$V \rho_i g = V_i \rho_w g$

$\frac{V}{V_i} = \frac{\rho_w}{\rho_i}$

$H_i = 0.9\ H$

So height inside water

$H_i = 0.9 \times 8 = 7.2$ m

So height ouside water

$h_0 \simeq 0.8$ m

Nearest value in 0.9 m

18. A box filled with water has a small hole on its side near the bottom. It is dropped from the top of a tower. As it falls, a camera attached on the side of the box records the shape of the water stream coming out of the hole. The resulting video will show

(A) the water coming down forming a parabolic stream

(B) the water going up forming a parabolic stream

(C) the water coming out in a straight line

(D) no water coming out

Ans. **[D]**

Sol. By theory

19. An earthen pitcher used in summer cools water in it essentially by evaporation of water from its porous surface. If a pitcher carries 4 kg of water and the rate of evaporation is 20 g per hour, temperature of water in it decreases by T in two hours. The value of T is close to (ratio of latent of evaporation to specific heat of water is 540 ºC)

(A) 2.7 ºC (B) 4.2 ºC (C) 5.4 ºC (D) 10.8 ºC

Ans. **[C]**

Sol. $Q_{Heating} = MS\ Q$

$Q_{Evaporation} = (\ m)\ L$

$Q_{Heating} = Q_{evaporation}$

$MS\ Q = (\ m)\ L$

$$ms\frac{dQ}{dt} = \frac{dm}{dt}L$$

$$\frac{dQ}{dt} = \frac{L}{ms}\frac{dm}{dt}$$

$$= \frac{L/s}{m}\frac{dm}{dt}$$

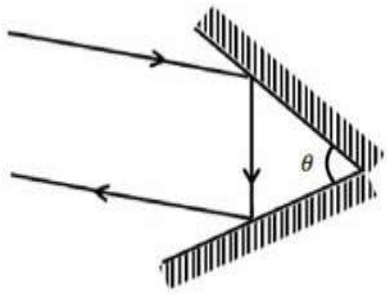
$$= \frac{540}{4h} \times 20 \times 10^{-3} \frac{°C}{°C}$$

$$\frac{dQ}{dt} = 2.7\ \overline{h}$$

$T = 2.7 \times 2$

$= 5.4\ °C$

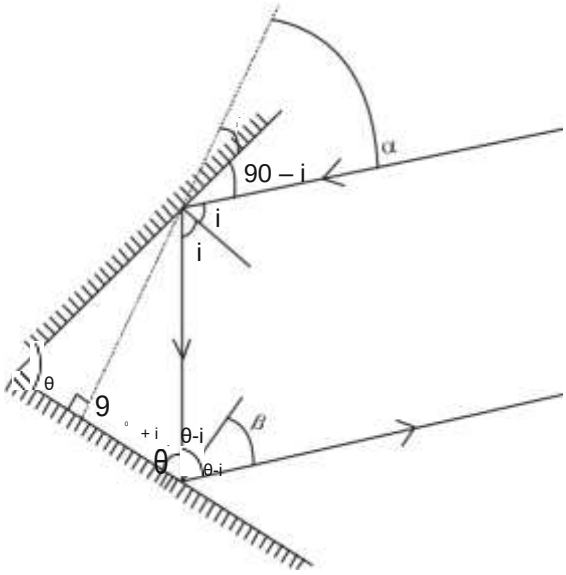
20. Two plane mirrors are kept on a horizontal table making an angle θ with each other as shown schematically in the figure. The angle θ is such that any ray of light reflected after striking both the mirrors returns paralled to its incident path. For this to happen, the value of q should be



(A) 30º (B) 45º (C) 60º (D) 90º

Ans. **[D]**

Sol.



$\alpha = \beta$

$90 - \theta + 90 - i = \theta - i$

$180 - \theta - i = \theta - i$

$2\theta = 180$

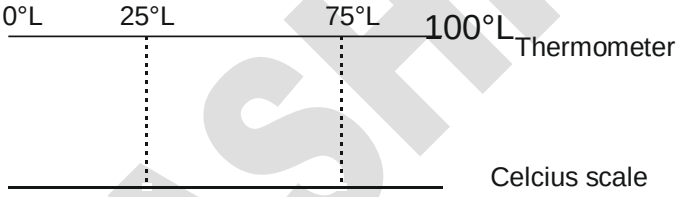
$\theta = 90$

21. A certain liquid has a melting point of –50º C and a boiling point of 150ºC. A thermometer is designed with this liquid and its melting and boiling points are designated as 0º L and 100ºL. The melting and boiling points of water on this scale are

(A) 25º L and 75º L, respectively (B) 0º L and 100º L, respectively

(C) 20º L and 70º L, respectively (D) 30º L and 80º L, respectively

Ans. [A]



Sol. –50°C 0°C 100°C 150°C

0°L – 100°L → 100 parts

–50°C – 150°C → 200 parts

1°L = 2°C

22. One can define an alpha–Volt (α *V*) to be the energy acquired by an α particle when it is accelerated by a potential of 1 Volt. For this problem you may take a proton to 2000 times heavier than an electron. Then

(A) 1 α V = 1 eV/4000 (B) 1 α V = 2 eV (C) 1 α V = 8000 eV (D) 1 α V = 1 eV

Ans. [B]

Sol. $k = q\,(v)$

$q_\alpha = 2q_e = 2e$

$k_\alpha = 2\,ke$

$1\alpha V = 2eV$

23. In a particle accelerator, a current of 500 μA is carried by a proton beam in which each proton has a speed of 3×10^7 m/s. The cross seetional area of the beam is 1.50 mm^2. The charge density in this beam in Coulomb/m^3 is close to–

(A) 10^{-8} (B) 10^{-7} (C) 10^{-6} (D) 10^{-5}

Ans. **[D]**

Sol. $i = ne\,A\,v$

$$\text{change density} = \frac{q}{V} = ne$$

$$ne = \frac{i}{Av} = \frac{500\times10^{-6}}{1.5\times10^{-6}\times3\times10^{7}} = 10^{-5}\,\frac{c}{m^3}$$

24. Which of the following is NOT true about the total lunar eclipse?

(A) A lunar eclipse can occur on a new moon and full moon day.

(B) The lunar eclipse would occur roughly every month if the orbits of earth and moon were perfectly coplanar

(C) The moon appear red during the eclipse because the blue light is absorbed in earth's atmosphere and red is transmitted.

(D) A lunar eclipse can occur only on a full moon day

Ans. **[A]**

Sol. By theory

25. Many exoplanets have been discovered by the transit method, wherein one monitors a dip in the intensity of the parent star as the exoplanet moves in front it. The exoplanet has a radius R and the parent star has radius 100R. If I_0 is the intensity observed on earth due to the parent star, then as the exoplanet transits.

(A) the minimum observed intensity of the parent star is 0.9 I_0

(B) the minimum observed intensity of the parent star is 0.99 I_0

(C) the minimum observed intensity of the parent star is 0.999 I_0

(D) the minimum observed intensity of the parent star is 0.9999 I_0

Ans. **[D]**

Sol. Minimum observed intensity

$$I = I_0 - \frac{A_P}{A_S} I_0$$

A_p = effective area of planet

$$I = I_0\left(1 - \frac{R^2}{(100R)^2}\right)$$

A_s = effective area of star

$$I = I_0\,\frac{9999}{10000}$$

$$I = 0.9999\ I_0$$

26. A steady current I is set up in a wire whose cross-sectional area decreases in the direction of the flow of the current. Then, as we examine the narrowing region

(A) the current density decreases in value

(B) the magnitude of the electric field increases

(C) the current density remains constant

(D) the average speed of the moving charges remains constant

$i = ne\,a\,v_d$

$v_d \propto \frac{1}{eE^{A}}$

$v_d = \frac{}{m}$

$v_d \propto E$

—

27. Select the correct statement about rainbow

(A) We can see a rainbow in the western sky in the late afternoon

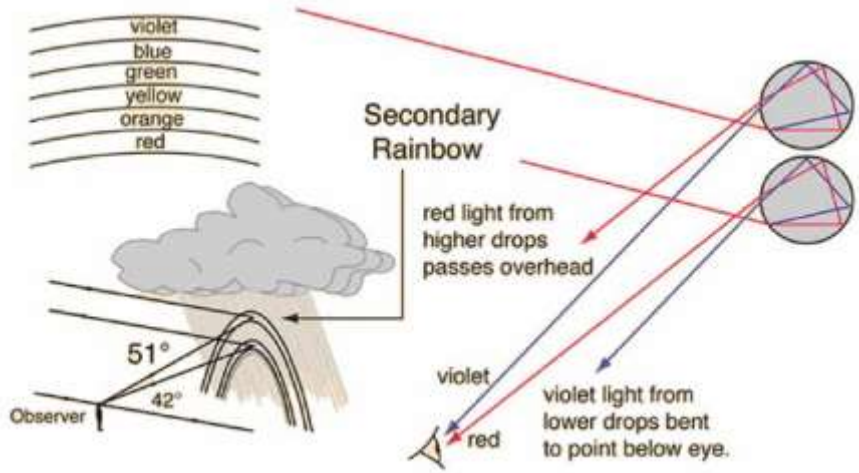
(B) The double rainbow has red on the inside and violet on the outside

(C) A rainbow has an arc shape since the earth is round

(D) A rainbow on the moon is violet on the inside and red on the outside

Ans. [B]

Sol.



28. Remote sensing satellites move in an orbit that is at an average height of about 500 km from the surface of the earth. The camera onboard one such satellite has a screen on area A on which the images captured by it are formed. If the focal length of the camera lens is 50 cm, then the terrestrial area that can be observed from the satellite is close to

(A) 2×10^3 A (B) 10^6 A (C) 10^{12} A (D) 4×10^{12} A

Ans. [C]

Sol. $A = m^2 A = \frac{v_2}{u^2} A$

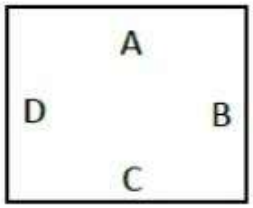
$v = 50\ cm = 50 \times 10^{-2}\ m$

$u = 500\ km = 500 \times 10^3\ m$

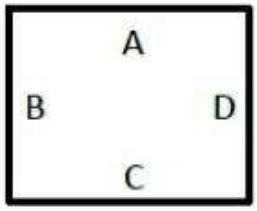
$A_i = A$

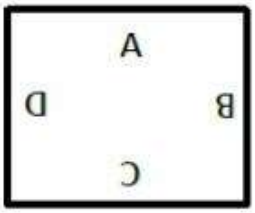
$A_0 = \frac{A_i}{m^2} \frac{A}{\frac{50 \times 10^{-2}}{500 \times 10^3}} = 10^{12} A$

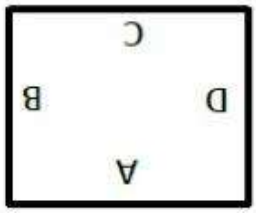
29. Letters A, B, C and D are written on a cardboard as shown in the picture.

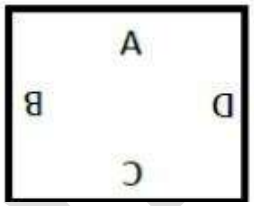


The cardboard is kept at a suitable distance behind a transparent empty glass of cylindrical shape. If the glass is now filled with water, one sees an inverted image of the pattern on the cardboard when looking through the glass. Ignoring magnification effects, the image would appear as

(A) 

(B) 

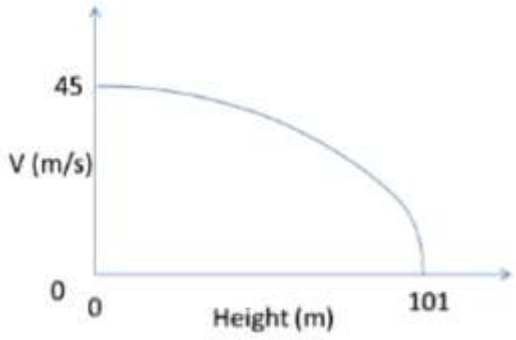
(C) 

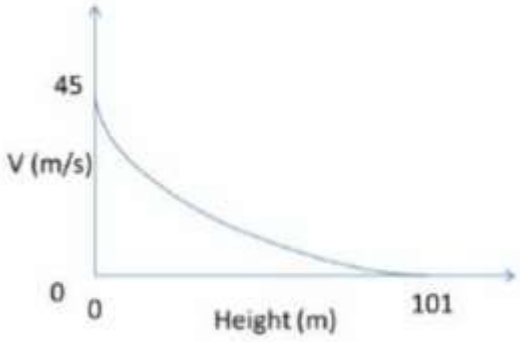
(D) 

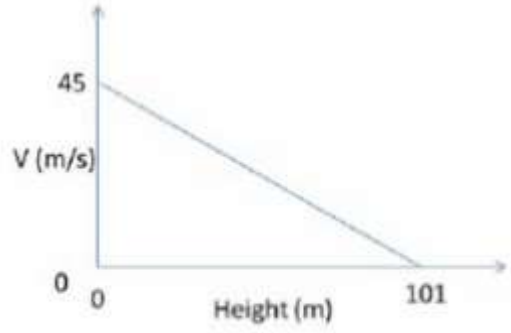
Ans. **[D]**

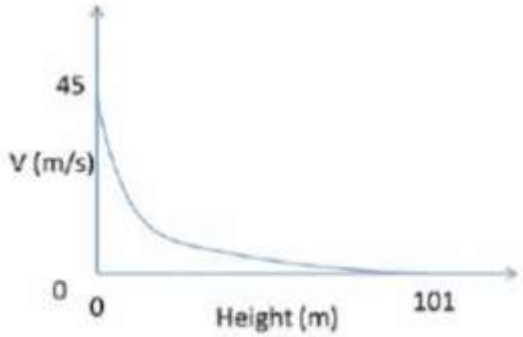
Sol. Mirror Image

30. If a ball is thrown at a velocity of 45 m/s in vertical upward direction, then what would be the velocity profile as function of height? Assume g = 10 m/s^2.

(A) 

(B) 

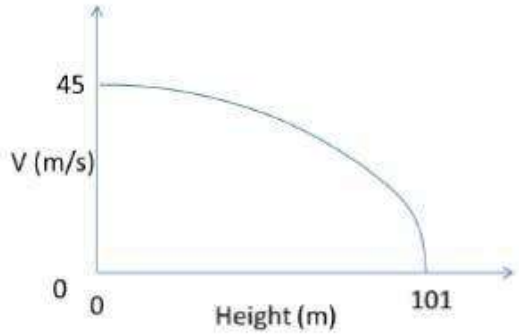
(C) 

(D) 

45_2

$h = \frac{}{} = 101.25 \simeq 101$ m

$h \propto v^2$



CHEMISTRY

31. The number of water molecules in 250 mL of water is closest to

[Given : Density of water is 1.0 g mL^{-1} ; Avogadro's number = 6.023×10^{23}]

(A) 83.6×10^{23} (B) 13.9×10^{23} (C) 1.5×10^{23} (D) 33.6×10^{23}

Ans. [A]

Sol. Density of water = 1g/mL

So 250 mL = 250 g water

Moles of $H_2O = \frac{250}{18}$

No. of water molecules = $\frac{250}{18} \times 6.023 \times 10^{23}$

$= 83.6 \times 10^{23}$

32. Among the following, the correct statement is

(A) pH decreases when solid ammonium chloride is added to a dilute aqueous solution of NH_3

(B) pH decreases when solid sodium acetate is added to dilute aqueous solution of acetic acid

(C) pH decreases when solid NaCl added to a dilute aqueous solution of NaOH

(D) pH decreases when solid sodium oxalate is added to a dilute aqueous solution of oxalic acid

Ans. [A]

Sol.dil. aq. sol^n of $NH_3 = NH_4OH$

$NH_4OH \rightleftharpoons NH_4^+ + OH^-$

$NH_4Cl \rightarrow NH_4^+ + Cl^-$

due to common ion effect, equilibrium of NH_4OH shifts in backward direction so $[OH^-] \downarrow$

$[OH^-] \propto \frac{1}{pOH}$ then pOH ↑ pH ↓

33. The solubility of $BaSO_4$ in pure water (in g L^{-1}) is closest to
[Given : K_{sp} for $BaSO_4$ is 1.0×10^{-10} at 25 °C. Molecular weight of $BaSO_4$ is 233 g mol^{-1}]
(A) 1.0×10^{-5} (B) 1.0×10^{-3} (C) 2.3×10^{-5} (D) 2.3×10^{-3}

Ans. **[D]**

Sol. $K_{sp}(BaSO_4) = s^2$

$s = \sqrt{k_{sp}} = \sqrt{1 \times 10^{-10}} = 10^{-5}$ molL^{-1}

Solubility $s = 10^{-5} \times$ molecular mass
$= 10^{-5} \times 233$ g L^{-1}
$= 2.33 \times 10^{-3}$ g L^{-1}

34. Among the following, the **INCORRECT** statement is
(A) No two electrons in an atom can have the same set of four quantum numbers
(B) The maximum number of electrons in the shell with principal quantum number, n, is equal to n^2+2
(C) Electrons in an orbital must have opposite spin
(D) In the ground state, atomic orbitals are filled in the order of their increasing energies

Ans. **[B]**

Sol. The maximum number of e^- in the shell with principal quantum number n, is equal to $2n^2$ (not $n^2 + 2$)

35. A container of volume 2.24 L can withstand a maximum pressure of 2 atm at 298 K before exploding. The maximum amount of nitrogen (in g) that can be safely put in this container at this temperature is closest to
(A) 2.8 (B) 5.6 (C) 1.4 (D) 4.2

Ans. **[B]**

Sol. Ideal gas equation $PV = nRT$

Moles of N_2 gas (n) $= \frac{PV}{RT} = \frac{2\text{atm} \times 2.24\text{L}}{0.821\text{Latm mol}^{-1}\text{K}^{-1} \times 298\text{K}}$

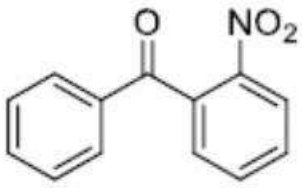
$n = \frac{4.48}{0.0821 \times 298}$

Mass of N_2 gas (in g) = moles × mol. Mass

$= \frac{4.48}{0.0821 \times 298} \times 28$

$= 5.6$ g

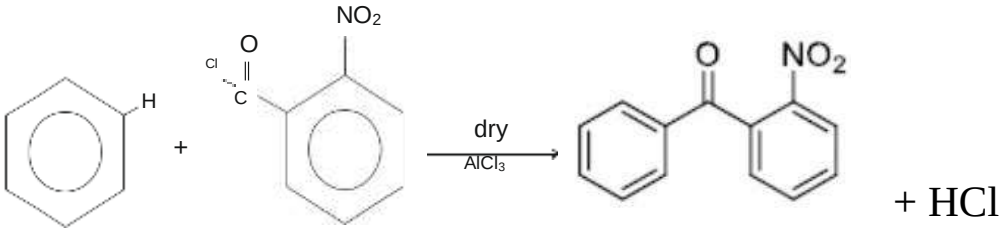
36. The compound shown below



Can be readily prepared by Friedel-Crafts reaction between
(A) benzene and 2-nitrobenzoyl chloride
(B) benzyl chloride and nitrobenzene
(C) nitrobenzene and benzoyl chloride
(D) benzene and 2-nitrobenzyl chloride

Ans. **[A]**

Sol.



+ HCl

Benzene gives F.C.R. due to its nucleophilic nature.

37. The correct statement about the following compounds is

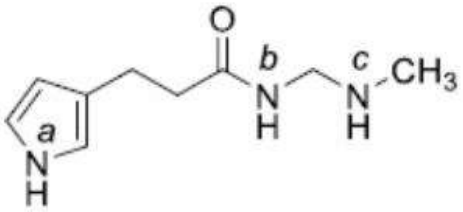
X Y

(A) both are chiral
(B) both are achiral
(C)X is chiral and Y is achiral
(D) X is achiral and Y is chiral

Ans. **[C]**

Sol. 'X' is chiral molecule because it has chiral centre or asymmetric C atom but 'Y' is achiral because it does not have chiral centre.

38. The most acidic proton and the strongest nucleophilic nitrogen in the following compound



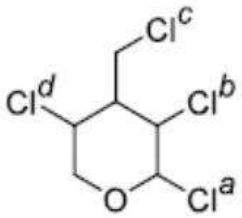
Respectively, are

(A) N^a–H; N^b (B) N^b–H; N^c (C) N^a-H; N^c (D) N^c-H; N^a

Ans. **[B]**

Sol. –C–NH– group is amphoteric in nature so it is more acidic than ordinary amines.
‖
O

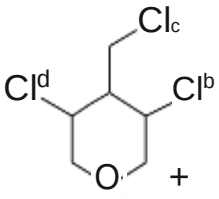
39. The chlorine atom of the following compound



That reacts most readily with $AgNO_3$ to give a precipitate is

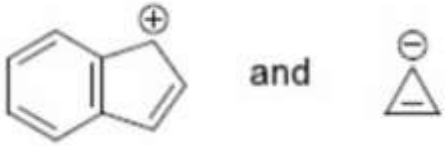
(A) Cl^a (B) Cl^b (C) Cl^c (D) Cl^d

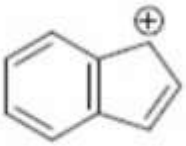
Ans. **[A]**

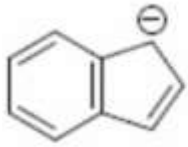
Sol. Its 

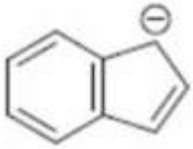
carbocation is very stable due to vacant orbital resonance.

40. Among the following sets, the most stable ionic species are

(A)  and

(B)  and 

(C)  and 

(D)  and 

Ans. **[D]**

Sol. Given ions in option (D) are aromatic in nature therefore they are highly stable.

41. The correct order of energy of 2s orbitals in H, Li, Na and K, is

(A) K < Na < Li < H (B) Na < Li < K < H (C) Na < K < H < Li (D) H < Na < Li < K

Ans. **[A]**

Sol. As we go down the group IA, there is increase in shell so size of atom increases and energy of 2s orbital decreases.

42. The hybridization of xenon atom in XeF_4 is

(A) sp^3 (B) dsp^3 (C) sp^3d^2 (D) d^2sp^3

Ans. **[C]**

Sol. In XeF_4, central atom Xe contains 2 lone pair and 4 bond pair [σ bond]

so steric number = no. of l p + no. of σ bond

= 2+4=6

Thus hybridization is sp^3d^2

43. The formal oxidation numbers of Cr and Cl in the ions $Cr_2O_7^{2-}$ and ClO_3^- respectively, are

(A) +6 and +7 (B) +7 and +5 (C) +6 and +5 (D) +8 and +7

Ans. **[C]**

Sol. Let O.N. of Cr in $Cr_2O_7^{2-}$ is x

sum of oxidation no. of all atoms in an ion is equal to charge present on ion.

$2x + 7(-2) = -2$

$2x + 14 = -2$

$x = +6$

Let O.N. of Cl in ClO_3^- is x

$x + 3(-2) = -1$

$x = +5$

44. A filter paper soaked in salt X turns brown when exposed to HNO_3 vapor. The salt X is –

(A) KCl (B) KBr (C) KI (D) K_2SO_4

Ans. **[C]**

Sol. Filter paper soaked with KI turns brown when exposed to HNO_3 vapour due to liberation of iodine. The reaction is as follows :

$6KI + 8HNO_3 \rightarrow 6KNO_3 + 4H_2O + 2NO + 3I_2$

45. The role of haemoglobin is to

(A) store oxygen in muscles (B) transport oxygen to different parts of the body

(C) convert CO to CO_2 (D) convert CO_2 into carbonic acid

Ans. **[B]**

Sol. The role of haemoglobin is to transport oxygen to different parts of the body.

BIOLOGY

46. Which ONE of the following molecules is a secondary metabolite?

(A) Ethanol (B) Lactate (C) Penicillin (D) Citric acid

Ans. **[C]**

Sol. Molecules, which cannot a directly used and synthesized by animal body but may changes primary process of living system called as secondary metabolite.

Penicillin is an antibiotic optained from penicilium notatum used for treatment bacterial infection that provide artificial passive acquired immunity.

47. Lecithin is a

(A) carbohydrate (B) phospholipid (C) nucleoside (D) protein

Ans. **[B]**

Sol. It is called phospholipid molecule consist of two fatty acids, one glycerol, one phosphate and one choline molecule, it is also called phosphatidyl choline which is amphipathic molecule or membrane lipid forming plasma membrane of the cell.

48. The water potential (ψp) of pure water at standard temperature and atmospheric pressure is

(A) 0 (B) 0.5 (C) 1.0 (D) 2.0

Ans. **[A]**

Sol. ψ_w of pure water is zero at standard temperature & atmospheric pressure because there is no solute in pure water so water potential (ψ_w) is zero.

49. Action potential in neurons is generated by a rapid influx of

(A) chloride ions (B) potassium ions (C) calcium ions (D) sodium ions

Ans. **[D]**

Sol. Sodium an influx into the axoplasm (three sodium ion) that stimulate neural conduction.

50. Erythropoietin is produced by

(A) heart (B) kidney (C) bone marrow (D) adrenal gland

Ans. **[B]**

Sol. Justa glomerular cells of kidney produce erythroprotein hormone during oxygen deficiency.

It promotes red bone marrow to form RBC (Erythropoisis).

51. Tendrils are modifications of

(A) stem or leaf (B) stem only (C) leaf only (D) aerial roots only

Ans. **[A]**

Sol. In plant tendril is a modification of stem, leaves or petiole with a thread like coiled shape. Function of tendril is for support, climbing, attachment, cellular invasion by parasitic plant.

Example : Leaves modified into tendril in pea for climbing.

Stem tendril which develop from axillary bud in cucumber, pumpkins, watermelon and grapevines for climbing.

52. Which ONE of the following combinations of biomolecules is present in the ribosomes?

(A) RNA, DNA and protein (B) RNA, lipids and DNA

(C) RNA and protein (D) RNA and DNA

Ans. **[C]**

Sol. Ribosome are granular structure smallest cell organelle. Composed of RNA & protein only.

53. Which ONE of the following proteins does NOT play a role in skeletal muscle contraction?

(A) Actin (B) Myosin (C) Troponin (D) Microtubule

Ans. **[D]**

Sol. For muscular contraction actin, myosin (contractile protein) and troponin, tropomyosin (regulatory protein) essential for muscular contraction.

54. Which ONE of the following reactions is catalyzed by high-energy ultraviolet radiation in the stratosphere?

(A) $O_2+O \rightarrow O_3$ (B) $O_2 \rightarrow O+O$ (C) $O_3+O_3 \rightarrow 3O_2$ (D) $O+O \rightarrow O_2$

Ans. **[B]**

Sol. Ozone formed naturally in stratosphere by including two step or two reaction.

I. Ist reaction catalyzed by high energy ultraviolet radiation. O_2

$\xrightarrow{UV} O+O$

II. $O_2 \rightarrow O+O_3$

55. Which ONE of the following statements is TRUE about trypsinogen?

(A) It is activated by enterokinae (B) It is activated by renin

(C) It is activated by pepsin (D) It does not need activation

Ans. **[A]**

Sol. Trypisonogen is an inactive enzyme produced from pancreatic juice, which get activated by the action of enterokinase produced from intestinal gland in small intestine into its active form trypsin.

56. Which ONE of the following organisms respire through the skin?

(A) Blue whale (B) Salamander (C) Platypus (D) Peacock

Ans. **[B]**

Sol. Direct example.

57. Which ONE of the following humans cells lacks a nucleus?

(A) Neutrophil (B) Neuron (C) Mature erythrocyte (D) Keratinocyte

Ans. **[C]**

Sol. Prokaryotic cell, mature erythrocyte (animal) mature sieve tube cell of phloem are example of nucleus less cell.T

58. The first enzyme that the food encounters in human digestive system is

(A) pepsin (B) trypsin (C) chymotrypsin (D) amylase

Ans. **[D]**

Sol. In saliva α amylase is required for intinal first digestion of starch in oral cavity.

59. Glycoproteins are formed in which ONE of the following organelles?

(A) Peroxisome (B) Lysosome (C) Golgi apparatus (D) Mitochondria

Ans. **[C]**

Sol. Golgi apparatus is the important site of formation of glycoprotein & glycolipid.

60. An example of nastic movement (external stimulus-dependent movement) in plants is

(A) folding-up of the leaves of *Mimosa pudica*

(B) climbing of tendrils

(C) growth of roots from seeds

(D) growth of pollen tube towards the ovule

Ans. **[A]**

Sol. Nastic movement are non directional movement which is depend on external stimules in plant.

This movement can be due to changes in turgor or changes in growth.

Example : Folding of leaf of mimosa pudica by touching.

Diurnal movement of leaves

Response of insectivorous plant such as venus fly trap.

Part – II

Two - Mark Questions

MATHEMATICS

61. What is the sum of all natural number n such that the product of the digits of n (in base 10) is equal to $n^2 - 10n - 36$?

(A) 12 (B) 13 (C) 124 (D) 2612

Ans. **[B]**

Sol. let product of digits of n be p(n)

Note : $p(n) \le n$

let $n = a_m 10^m + a_{m-1} 10^{m-1} + \ldots\ldots\ldots\ldots + a_0$

$\ge a_m 10^m$

$\ge a_m 9^m$

$\ge a_m a_{m-1} \overset{\ldots\ldots\ldots\ldots\ldots}{} a_0$

$= p(n)$

Now $n^2 - 10n - 36 \le n$

$n^2 - 11n - 36 \le 0$

$-2.64 \le n \le 13.64$

$\Rightarrow -3 < n < 14$ ……..(i)

Also $p(n) \ge 0$

$n^2 - 10n - 36 \ge 0$

$(n-5)^2 \ge 61$

$n \ge 12.81$ ……..(ii)

from (i) & (ii)

$n = 13$

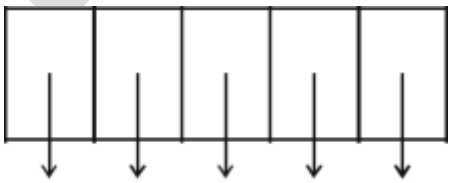
62. Let m (respectively, n) be the number of 5-digit integers obtained by using the digits 1,2,3,4,5 with repetitions (respectively, without repetitions) such that the sum of any two adjacent digits is odd. Then $\frac{m}{n}$ is equal to

(A) 9 (B) 12 (C) 15 (D) 18

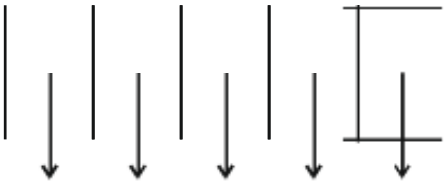
Ans. **[C]**

Sol. When repetition is allowed

m =



(even)



(odd)

$2\times3\times2\times3\times2=72$ + $3\times2\times3\times2\times3=108$

63. The number of solid cones with integer radius and height each having its volume numerically equal to its total surface area is

(A) 0 (B) 1 (C) 2 (D) infinite

Ans. **[B]**

Sol. r & h are integers

$\frac{1}{3}\pi r^2 h = \pi r l + \pi r^2 \quad (l = \sqrt{r^2 + h^2})$

$rh - 3r = 3\sqrt{r^2 + h^2}$

squaring both sides

$r^2h^2 + 9r^2 - 6hr^2 = 9r^2 + 9h^2$

$r^2(h - 6) = 9h$

$r^2 = \frac{9h}{h - 6}$

$r^2 = 9 + \frac{54}{h - 6}$

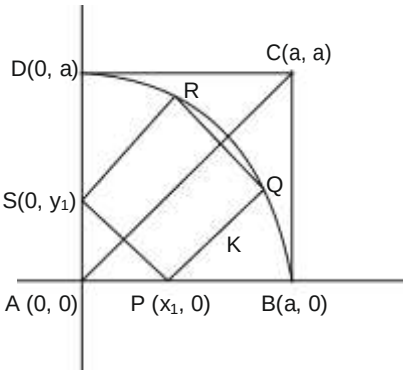
here r should be integer for which there exist only one solution which satisfy above condition is h = 8, and r = 6.

64. Let ABCD be a square. An arc of a circle with A as center and AB as radius is drawn inside the square joining the points B and D. Points P on AB, S on AD, Q and R on arc BD are taken such that PQRS is a square. Further suppose that PQ and RS are parallel to AC. Then $\frac{\text{area PQRS}}{\text{area ABCD}}$ is

(A) $\frac{1}{8}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$

Ans. **[D]**

Sol.



eq. of arc : $x^2 + y^2 = a^2$

k = length of side of PQRS

$Q = (x_1 + k\cos 45°, 0 + k\sin 45°)$

$Q = x_1 + \frac{k}{\sqrt{2}}, \frac{k}{\sqrt{2}}$

$R = (0 + k\cos 45°, y_1 + k\sin 45°)$

$R = \frac{k}{\sqrt{2}}, y_1 + \frac{k}{\sqrt{2}}$

Q and R lies on Arc BD

$\Rightarrow \quad x_1 + \frac{k}{\sqrt{2}}^2 + \frac{k^2}{2} = a^2$

$x_1^2 + \frac{k^2}{2} + \sqrt{2}kx_1 + \frac{k^2}{2} = a^2$ …….. (1)

Similarly $y_1^2 + \frac{k^2}{2} + \sqrt{2}ky_1 + \frac{k^2}{2} = a^2$ …….. (2)

And PS = k $\Rightarrow x_1^2 + y_1^2 = k^2$

Slope of AC = 1

$\Rightarrow$ slope of PS = –1

$\frac{-y_1}{x_1} = -1 \Rightarrow y_1 = x_1$

and $x_1^2 + y_1^2 = k^2 \Rightarrow x_1 = \frac{k}{\sqrt{2}} = y_1$

Now from (i)

$\frac{k^2}{2} + \sqrt{2} \cdot k \cdot \frac{k}{\sqrt{2}} = a^2 - k^2 \rightarrow k^2 = \frac{2}{5}a^2$

$\Rightarrow \frac{\text{area PQRS}}{2 \text{ area ABCD}} = \frac{k^2}{a^2} = \frac{2}{5}$

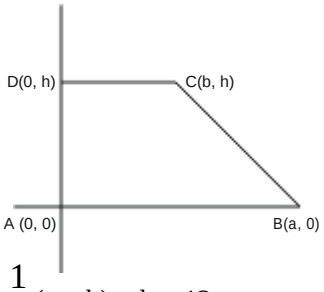
65. Suppose ABCD is a trapezium whose sides and height are integers and AB is parallel to CD. If the area of ABCD is 12 and the sides are distinct, then |AB – CD|

(A) is 2 (B) is 4

(C) is 8 (D) cannot be determined from the data

Ans. [B]

Sol.



$\frac{1}{2}(a + b) \times h = 12$

$a + b = \frac{24}{h}$

$BC = \sqrt{(a - b)^2 + h^2}$

$h = 3, a = 6, b = 2$

(so that $BC = \sqrt{(6 - 2)^2 + 3^2} = 5$)

$|a - b| = 4$

PHYSICS

66. A coffee maker makes coffee by passing steam through a mixture of coffee powder, milk and water. If the steam is mixed at the rate of 50 g per minute in a mug containing 500 g of mixture, then it takes about t_0 seconds to make coffee at 70 ºC when the initial temperature of the mixture is 25 ºC. The value of t_0 is close to (ratio of latent heat of evaporation to specific heat of water is 540 ºC and specific heat of the mixture can be taken to be the same as that of water)

(A) 30 (B) 45 (C) 60 (D) 90

Ans. [B]

Sol. $Q_{given} = Q_{absorbed}$

$M_m s\ Q_m = M_s L + M_s s\ Q_s$

$M_m s \frac{Q_m}{t} = \frac{M_s}{t} [L + s\ Q_s]$

$\frac{Q_m}{t} = \frac{M_s / t}{M_m} \frac{L}{s} + Q_s$

$\frac{45}{t} = \frac{50}{500}[540 + 30]$

$t = \frac{45}{57}$ minutes

$t = \frac{45}{57} \times 60$ seconds

67. A person in front of a mountain is beating a drum at the rate of 40 per minute and hears no distinct echo. If the person moves 90 m closer to the mountain, he has to beat the drum at 60 per minute to not hear any distinct echo. The speed of sound is

(A) 320 ms^{-1} (B) 340 ms^{-1} (C) 360 ms^{-1} (D) 380 ms^{-1}

Ans. **[C]**

Sol. For not hearing echo time interval between the beats of drum must be equal to time of echo

$$t_1 = \frac{2d}{v} = \frac{60}{40} = 3/2 \Rightarrow 2d = \frac{3}{2}v$$

$$t_2 = \frac{2(d-90)}{v} = \frac{60}{60} = 1$$

$$2d - 180 = v$$

$$\frac{3}{2}v - 180 = v$$

$$\frac{1}{2}v - 180$$

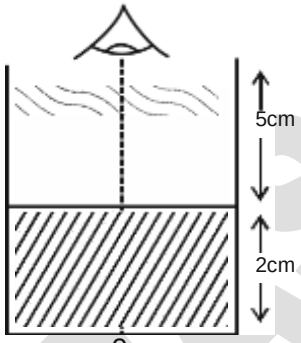
$$v \Rightarrow 360 m/s$$

68. A glass beaker is filled with water up to 5 cm. It is kept on top of a 2 cm thick glass slab. When a coin at the bottom of the glass slab is viewed at the normal incidence from above the beaker, it apparent depth from the water surface is d cm. Value of d is close to (the refractive indices of water and glass are 1.33 and 1.50, respectively)

(A) 2.5 (B) 5.1 (C) 3.7 (D) 6.0

Ans. **[B]**

Sol.



$$d = \frac{5cm}{\mu_w} + \frac{2cm}{\mu_g}$$

$$d = \frac{5}{4/3} + \frac{2}{3/2}$$

$$d = \frac{15}{4} + \frac{4}{3} =$$

$$12\frac{61}{} \quad D = 5.1 \text{ cm}$$

69. A proton of mass m and charge e is projected from a very larger distance towards an α particle with velocity *v*. Initially, α particle is at rest, but it is free to move. If gravity is neglected, then the minimum separation along the straight line of their motion will be

(A) $e^2/4\pi\in_0 mv^2$ (B) $5e^2/4\pi\in_0 mv^2$ (C) $2e^2/4\pi\in_0 mv^2$ (D) $4e^2/4\pi\in_0 mv^2$

Ans. [B]

Sol. From conservation of momentum

$mV = mV` + 4mV`$

$V=5V`$

$V` = V/5$

By energy conservation

$$\frac{1}{2}mv^2 = \frac{2K(e)^2}{r} + \frac{1}{2}m\frac{V^2}{5} + \frac{1}{2}4m\frac{V^2}{5}$$

$$\frac{1}{2}mv^2 = \frac{2K(e)^2}{r} + \frac{5}{2}m\frac{V^2}{5}$$

$$\frac{1}{2}mv^2 - \frac{1}{10}mv^2 = \frac{2e^2}{4\pi\in_0 r}$$

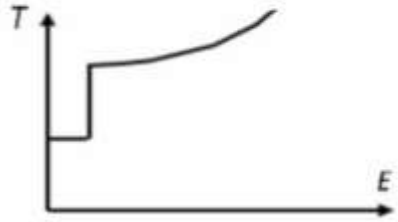
$$mv^2\frac{5-1}{10} = \frac{e^2}{2\pi\in_0 r}$$

$$\frac{2}{5}mv^2 = \frac{e^2}{2\pi\in_0 r}$$

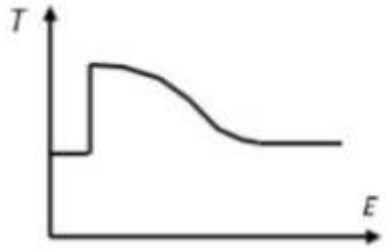
$$r = \frac{5e^2}{4\pi\in_0 v^2 m}$$ distance of closest approach

70. A potential is given by $V(x) = k(x + a)^2/2$ for $x < 0$ and $V(x) = k(x – a)^2/2$ for $x > 0$. The schematic variation of oscillation period (T) for a particle performing periodic motion in this potential as a function of its energy E is

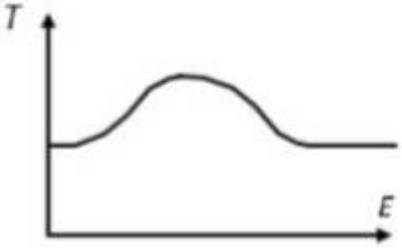
(A)



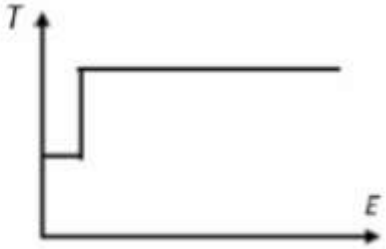
(B)



(C)

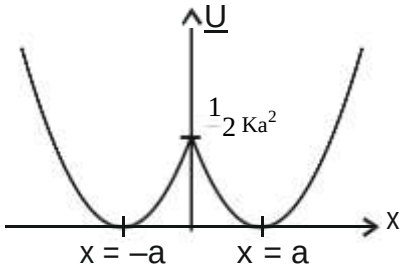


(D)



Ans. [B]

Sol.



For small energy the particle oscillates in one well when energy increases beyond $\frac{1}{2}Ka^2$ the oscillations are in both well so time period doubles. For very high value of energy the oscillations are such that time period again becomes equal to initial value.

CHEMISTRY

71. Among the following, the species with identical bond order are

(A) CO and O_2^{2-} (B) O_2^{2-} and CO (C) O_2^{2-} and B_2 (D) CO and N_2^+

Ans. **[C]**

Sol. O_2^{2-} contains 18 electrons same as F_2 so bond order = 1

$B_2 \rightarrow \sigma_{1s}^2, \sigma^*_{1s}{}^2, \sigma_{2s}^2, \sigma^*_{2s}{}^2, \pi_{2px}^1 = \pi_{2py}^1$

Bond order = $\frac{1}{2}(N_b - N_a)$

$= \frac{1}{2}(6-4)=1$

Thus O_2^{2-} and B_2 have identical bond order.

72. The quantity of heat (in J) required to raise the temperature of 1.0 kg of ethanol from 293.45 K to the boiling point and then change the liquid to vapour at that temperature is closest to

[Given : Boiling point of ethanol 351.45 K

Specific heat capacity of liquid ethanol 2.44 J g^{-1} K^{-1}

Latent heat of vaporization of ethanol 855 J g^{-1}]

(A) 1.42×10^2 (B) 9.97×10^2 (C) 1.42×10^5 (D) 9.97×10^5

Ans. **[D]**

Sol. Ethanol $\rightarrow$ Ethanol $\rightleftharpoons$ Ethanol

(l) (l) (g)

293.45K 351.45K

Heat required Q = ms T + mL (for phase change)

$Q = 10^3 \times 2.44 (351.45–293.45) + 10^3 (855)$

$Q = 10^3 [(2.44 \times 58) + 855] = 10^3 (996.52)$

$Q = 9.97 \times 10^5$

73. A solution of 20.2 of 1,2-dibromopropane in MeOH upon heating with excess Zn produce 3.58 g of an unsaturated compound X. The yield (%) of X is closest to
[Atomic weight of Br is 80]

(A) 18 (B) 85 (C) 89 (D) 30

Ans. [B]

Sol. $\underset{\underset{Br}{|}}{CH_2}\underset{\underset{Br}{|}}{CH} - CH_3 \xrightarrow[MeOH]{Zn} CH_2 = CH - CH_3$

mol. mass = 202 mol. mass = 42

moles = $\frac{20.2}{20.2} = 0.1$ moles = $\frac{3.58}{42}$

$= 0.085$

% yield = $\frac{0.085}{0.1} \times 100 = 85\%$

74. The lower stability of ethyl anion compared to methyl anion and the higher stability of ethyl radical compared to methyl radical, respectively, are due to

(A) +I effect of the methyl group in ethyl anion and $\sigma \rightarrow$ p-orbital conjugation in ethyl radical

(B) –I effect of the methyl group in ethyl anion and $\sigma \rightarrow \sigma^*$ conjugation in ethyl radical

(C) +I effect of the methyl group in both cases

(D) +I effect of the methyl group in ethyl anion and $\sigma \rightarrow \sigma^*$ conjugation in ethyl radical

Ans. [A]

Sol. Stability $C_2H_5^- > CH_3^-$

In ethyl anion, methyl group have +I effect which increases e^- density on carbanion so stability decreases.

Stability $C_2H_5 > CH_3$

Due to σ –p orbital conjugation in ethyl radical, it is more stable.

75. The F – Br-F bond angles in BrF_5 and the Cl-P-Cl bond angles in PCl_5 , respectively, are

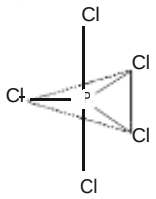
(A) identical in BrF_5 but non-identical in PCl_5 (B) identical in BrF_5 and identical in PCl_5

(C) non-identical in BrF_5 but identical in PCl_5 (D) non-identical in BrF_5 and non-identical in PCl_5

Ans. [D]

Sol.

BrF_5



PCl_5

Due to presence of lone pair on central atom, shape of BrF_5 becomes distorted so F–Br–F bond angles in BrF_5 are non identical

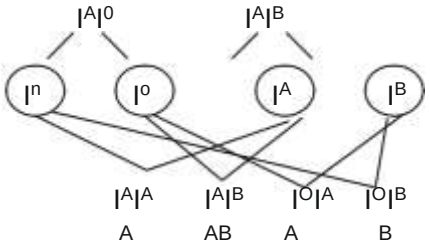
PCl_5 [shape → trigonal bipyramidal]
Cl–P–Cl bond angles = 120º & 90°
[sp^3d hybridisation]

76. If the genotypes determining the blood groups of a couple are I^AI^O and I^AI^B, then the probability of their first child having type O blood is

(A) 0 (B) 0.25 (C) 0.50 (D) 0.75

Ans. **[A]**

Sol.



No 'O' blood group probability

77. A cross was carried out between two individuals heterozygous for two pairs of genes was carred out. Assuming segregation and independent assortment, the number of different genotypes and phenotypes obtained respectively would be

(A) 4 and 9 (B) 6 and 3 (C) 9 and 4 (D) 11 and 4

Ans. **[C]**

Sol. Genotype = 3^n n = no. of heterozygous pair

$= 3^2=3\times3=9$

Phenotype = 2^n

$= 2^2=2\times2=4$

78. If the H^+ concentration of an aqueous solutions is 0.001 M, then the pOH of the solution would be

(A) 0.001 (B) 0.999 (C) 3 (D) 11

Ans. **[D]**

Sol. $[H^+]= 10^{-3}$ M

$pH = -\log_{10}[H^+]$

$= -\log_{10}[10^{-3}]$ pH = 3

pH + POH = 14

POH=14–3=11

79. Consider the following vision defects listed in Column I & II and the corrective measures in Column III. Choose the correct combination.

Column I	**Column II**	**Column III**
P. Hypermetropia	i. near-sightedness	a. convex lens
Q. Myopia	ii. Far-sightedness	b. concave lens

(A) P-ii-b (B) Q-i-b (C) P-i-a (D) Q-i-a

Ans. **[B]**

Sol. Direct answer

80. Which ONE of the following properties causes the plant tendrils to coil around a bamboo stick?

(A) Tendril has spines

(B) The base of the tendril grows faster than the tip

(C) Part of the tendril in contact with the bamboo stick grows at a slower rate than the part away from it

(D) The tip of the tendril grows faster than the base

Ans. [C]

Sol. Tendrils are sensitive to touch when they come in contact with the object does not grow rapidally as the part of the tendril away from the object. This cause the tendril to circle around the object & thus cling to it.