KVPY QUESTION PAPER-2019 (STREAM SB)

Part – I

One - Mark Questions **Date : 03 / 11 / 2019**

MATHEMATICS

1. The number of four-letter words that can be formed with letters a, b, c such that all three letters occur is

(A) 30 (B) 36 (C) 81 (D) 256

Ans. **[B]**

Sol. a, b, c

$${}^3C_1 \times \frac{4!}{2!} = 3 \times 12 = 36$$

2. Let $A = \left\{\theta \in R : \left(\frac{1}{3}\sin(\theta) + \frac{2}{3}\cos(\theta)\right)^2 = \frac{1}{3}\sin^2(\theta) + \frac{2}{3}\cos^2(\theta)\right\}$. Then

(A) $A \cap [0, \pi]$ is an empty set

(B) $A \cap [0, \pi]$ has exactly one point

(C) $A \cap [0, \pi]$ has exactly two points

(D) $A \cap [0, \pi]$ has more than two points

Ans. **[B]**

Sol. $\left(\frac{1}{3}\sin\theta + \frac{2}{3}\cos\theta\right)^2 = \frac{1}{3}\sin^2\theta + \frac{2}{3}\cos^2\theta$

$$\frac{1}{9}\sin^2\theta + \frac{4}{9}\cos^2\theta + \frac{4}{9}\sin\theta\cos\theta = \frac{1}{3}\sin^2\theta + \frac{2}{3}\cos^2\theta$$

$$\sin 2\theta = 1$$

$$2\theta = 2n\pi + \frac{\pi}{2}, n \in I$$

$$\theta = \frac{\pi}{4}, \theta \in [0, \pi]$$

$$A \cap [0, \pi] = \frac{\pi}{4}$$

3. The area of the region bounded by the lines x = 1, x = 2 and the curves $x(y - e^x) = \sin x$ and $2xy = 2\sin x + x^3$ is

(A) $e^2 - e - \frac{1}{6}$ (B) $e^2 - e - \frac{7}{6}$

(C) $e^2 - e + \frac{1}{6}$ (D) $e^2 - e + \frac{7}{6}$

Ans. **[B]**

Sol. $y = \frac{\sin x}{x} + e^x$ & $y = \frac{\sin x}{x} + \frac{x^2}{2}$

$$A = \int_1^2 \left\{\frac{\sin x}{x} + e^x - \left(\frac{\sin x}{x} + \frac{x^2}{2}\right)\right\} dx$$

$$= \int_1^2 \left(e^x - \frac{x^2}{2}\right) dx$$

$$= \left[e^x - \frac{x^3}{6}\right]_1^2$$

$$= e^2 - \frac{8}{6} - \left(e - \frac{1}{6}\right)$$

$$= e^2 - e - \frac{7}{6}$$

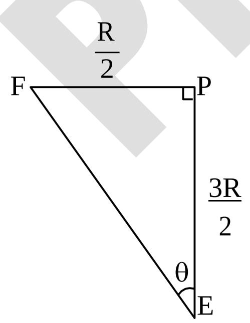
4. Let AB be a line segment with midpoint C, and D be the midpoint of AC. Let C_1 be the circle with diameter AB, and C_2 be the circle with diameter AC. Let E be a point on C_1 such that EC is perpendicular to AB. Let F be a point on C_2 such that DF is perpendicular to AB, and E and F lie on opposite sides of AB. Then the value of $\sin \angle FEC$ is

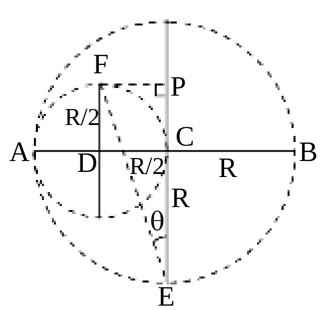
(A) $\frac{1}{\sqrt{10}}$ (B) $\frac{2}{\sqrt{10}}$ (C) $\frac{1}{\sqrt{13}}$ (D) $\frac{2}{\sqrt{13}}$

Ans. **[A]**

Sol. $\sin \angle FEC = \sin \theta$

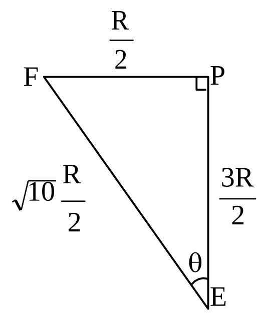
$\sin \theta$ **= ?**





$$FE^2 = \frac{R^2}{4} + \frac{9R^2}{4} = \frac{10R^2}{4}$$

$$FE = \frac{\sqrt{10}R}{2}$$



$$\sin\theta = \frac{\frac{R}{2}}{\sqrt{10}\frac{R}{2}}$$

$$\sin\theta = \frac{1}{\sqrt{10}}$$

5. The number of integers x satisfying $-3x^4 + \det\begin{bmatrix} 1 & x & x^2 \\ 1 & x^2 & x^4 \\ 1 & x^3 & x^6 \end{bmatrix} = 0$ is equal to

(A) 1 (B) 2 (C) 5 (D) 8

Ans. **[B]**

Sol. $-3x^4 + \begin{vmatrix} 1 & x & x^2 \\ 1 & x^2 & x^4 \\ 1 & x^3 & x^6 \end{vmatrix} = 0$

$$\Rightarrow -3x^4 + x^3 \begin{vmatrix} 1 & 1 & 1 \\ 1 & x & x^2 \\ 1 & x^2 & x^4 \end{vmatrix} = 0$$

$$\Rightarrow -3x^4 + x^3 \begin{vmatrix} 0 & 1-x & 1-x^2 \\ 0 & x-x^2 & x^2-x^4 \\ 1 & x^2 & x^4 \end{vmatrix} = 0 \qquad [R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3]$$

$$\Rightarrow -3x^4 + x^3 [x^2 (1-x^2)(1-x) - x(1-x)(1-x^2)] = 0$$

$$\Rightarrow x^4 [-3 + (1-x)(1-x^2)(x-1)] = 0$$

$x = 0, x = 2$

6. Let P be a non-zero polynomial such that $P(1 + x) = P(1 - x)$ for all real x, and $P(1) = 0$. Let m be the largest integer such that $(x - 1)^m$ divides P(x) for all such P(x). Then m equals

(A) 1 (B) 2 (C) 3 (D) 4

Ans. **[B]**

Sol. P(x) is non –zero polynomial and $P(1 + x) = P(1 - x)$ for all x

Differentiate w.r.t. x

$$P'(1 + x) = -P'(1 - x)$$

Put $x = 0$, $P'(1) = -P'(1) \Rightarrow P'(1) = 0$

and $P(1) = 0 \Rightarrow P(x)$ touch x-axis at $x = 1$

$\Rightarrow P(x) = (x - 1)^2 Q(x)$

$\Rightarrow m = 2$ such that $(x - 1)^m$ divides P(x) for all such P(x)

7. Let $f(x) = \begin{cases} x \sin\left(\dfrac{1}{x}\right) & \text{when } x \neq 0 \\ 1 & \text{when } x = 0 \end{cases}$ and $A = \{x \in R : f(x) = 1\}$. Then A has

(A) exactly one element (B) exactly two element

(C) exactly three element (D) infinitely many elements

Ans. **[A]**

Sol. $f(x) = \begin{cases} x \sin\left(\dfrac{1}{x}\right), & x \neq 0 \\ 1, & x = 0 \end{cases}$

$$x \sin\left(\frac{1}{x}\right) = 1$$

$$\sin\left(\frac{1}{x}\right) = \frac{1}{x}, \ x \neq 0$$

$$\because \sin\left(\frac{1}{x}\right) < \frac{1}{x}, \ x > 0$$

$$\text{and } \sin\left(\frac{1}{x}\right) > \frac{1}{x}, \ x < 0$$

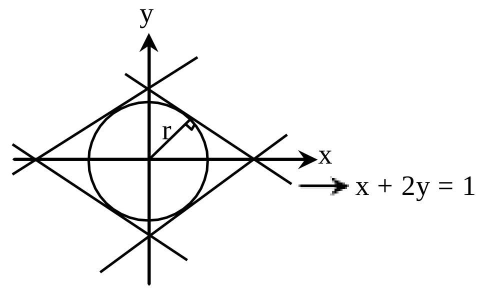
So only one solution

8. Let S be a subset of the plane defined by : $S = \{(x, y) : |x| + 2|y| = 1\}$. Then the radius of the smallest circle with centre at the origin and having non-empty intersection with S is

(A) $\dfrac{1}{5}$ (B) $\dfrac{1}{\sqrt{5}}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{2}{\sqrt{5}}$

Ans. **[B]**

Sol. S = {(x, y) : |x| + 2|y| = 1}



$$r = \left|\frac{0+2\times0-1}{\sqrt{1^2+2^2}}\right| = \frac{1}{\sqrt{5}}$$

9. The number of solutions of the equation sin (9x) + sin (3x) = 0 in the closed interval [0, 2π] is

(A) 7 (B) 13 (C) 19 (D) 25

Ans. **[B]**

Sol. $\sin(9x) + \sin(3x) = 0, \quad x \in [0, 2\pi]$

$3\sin 3x - 4\sin^3(3x) + \sin 3x = 0$

$\sin 3x\,(1 - \sin^2 3x) = 0$

$\sin 3x = 0,$ or $\sin^2 3x = 1$

$3x = n\pi\,,\ n\in I$ or $3x = k\pi + \frac{\pi}{2},\ k\in I$

$x = \frac{n\pi}{3}$ $\quad x = \frac{k\pi}{3} + \frac{\pi}{6},\ k\in I$

n = 0, 1, 2, 3, 4, 5, 6 k = 0, 1, 2, 3, 4, 5

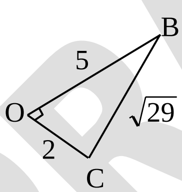
Total 13 solutions

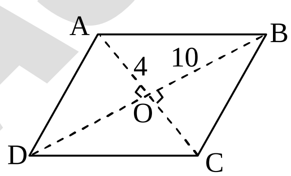
10. Among all the parallelograms whose diagonals are 10 and 4, the one having maximum area has its perimeter lying in the interval

(A) (19, 20] (B) (20, 21] (C) (21, 22] (D) (22, 23]

Ans. **[C]**

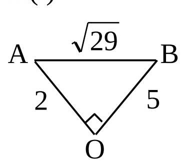
Sol.





For maximum Area it must be a Rhombus

$AD = BC = \sqrt{29}$ ...(i)



$AB = DC = \sqrt{29}$ ....(ii)

$AB = BC = CD = AD = \sqrt{29}$

$\text{Perimeter} = 4\sqrt{29} = \sqrt{16\times 29} = \sqrt{464} = 21.54$

Perimeter (21, 22]

11. The number of ordered pairs (a, b) of positive integers such that $\frac{2a-1}{b}$ and $\frac{2b-1}{a}$ are both integers is

(A) 1 (B) 2 (C) 3 (D) more than 3

Ans. **[C]**

Sol. (a, b) a, b, $\in I^+$

$\left(\frac{2a-1}{b}, \frac{2b-1}{a}\right) \in I^+$

Let $\frac{2a-1}{b} = 1$

$2a - 1 = b$

$\frac{2b-1}{a} = \frac{2(2a-1)-1}{a}$

$= \frac{4a-2-1}{a}$

$= \frac{4a-3}{a}$

$\left(\frac{2b-1}{a}\right) = \left(4 - \frac{3}{a}\right)$

For integer a = 1, 3

b = 1, 5

so total set (3)

(1, 1), (3, 5), (5, 3)

12. Let z = x + iy and w = u + iv be complex numbers on the unit circle such that $z^2 + w^2 = 1$. Then the number of ordered pairs (z, w) is

(A) 0 (B) 4 (C) 8 (D) infinite

Ans. **[C]**

Sol. $(x + iy)^2 + (u + iv)^2 = 1$

$x^2 - y^2 + u^2 - v^2 + 2i(xy + uv) = 1 + 0i$

$x^2 - y^2 + u^2 - v^2 = 1$ ..... (i) $xy + uv = 0$ (ii)

$x^2 + y^2 = 1$

$y^2 = 1 - x^2$

$v^2 = u^2 - 1$

Put into equation (i)

$x^2 + x^2 - 1 + u^2 + u^2 - 1 = 1$

$x^2 + u^2 = \frac{3}{2}$ ..... (iii)

equation (ii)

$xy + uv = 0$

$xy = -uv$

$x^2 y^2 = u^2v^2$

$x^2(1 - x^2) = u^2 (1 - u^2)$

From equation (iii) $u^2 = \frac{3}{2} - x^2$

$x^2 - x^4 = \left(\frac{3}{2} - x^2\right) - \left(\frac{3}{2} - x^2\right)^2$

$x^2 - x^4 = \frac{3}{2} - x^2 - \frac{9}{4} - x^4 + 3x^2$

$\frac{9}{4} - \frac{3}{2} = x^2$

$x^2 = \frac{3}{4}$

$x = \pm\frac{\sqrt{3}}{2}, y = \pm\frac{1}{2}$ By $(y^2 = 1 - x^2)$

$u^2 = \frac{3}{2} - \frac{3}{4}$

$u = \pm\frac{\sqrt{3}}{2}, v = \pm\frac{1}{2}$ By $(v^2 = 1 - u^2)$

Value of x and u Put in eq (ii)

$xy + uv = 0$

$x = \frac{\sqrt{3}}{2}$, $u = \frac{\sqrt{3}}{2} \Rightarrow \frac{\sqrt{3}}{2}y + \frac{\sqrt{3}}{2}v = 0$ $y = \frac{1}{2}$ then $v = -\frac{1}{2}$

2 solution $y + v = 0$ $y = -\frac{1}{2}$ then $v = \frac{1}{2}$

same as $x = +\frac{\sqrt{3}}{2}$ $u = -\frac{\sqrt{3}}{2}$ 2 solution

$x = -\frac{\sqrt{3}}{2}$ $u = \frac{\sqrt{3}}{2}$ 2 solution

$x = -\frac{\sqrt{3}}{2}$ $u = -\frac{\sqrt{3}}{2}$ 2 solution

Total = 8 solution of (z, w)

13. Let E denote the set of letters of the English alphabet, V = {a, e, i, o, u}, and C be the complement of V in E. Then, the number of four-letter words (where repetitions of letters are allowed) having at least one letter from V and at least one letter from C is

(A) 261870 (B) 314160 (C) 425880 (D) 851760

Ans. [A]

Sol.

V = {a, e, i, o, u}

C = E – V

Total words = $26 \times 26 \times 26 \times 26 - 5 \times 5 \times 5 \times 5 - 21 \times 21 \times 21 \times 21$

$= 261870$

14. Let σ_1, σ_2, σ_3 be planes passing through the origin. Assume that σ_1 is perpendicular to the vector (1, 1, 1), σ_2 is perpendicular to a vector (a, b, c), and σ_3 is perpendicular to the vector (a^2, b^2, c^2). What are all the positive values of a, b, and c so that $\sigma_1 \cap \sigma_2 \cap \sigma_3$ is a single point ?

(A) Any positive value of a, b, and c other than 1

(B) Any positive value of a, b, and c where either $a \neq b$, $b \neq c$ or $a \neq c$

(C) Any three distinct positive values of a, b, and c

(D) There exist no such positive real numbers a, b, and c

Ans. **[C]**

Sol. $\sigma_1 : x + y + z = 0$

$\sigma_2 : ax + by + cz = 0$

$\sigma_3 : a^2x + b^2y + c^2z = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

For unique solution, $\Delta \neq 0$

$\Delta = (a - b)(b - c)(c - a) \neq 0$

$a \neq b, b \neq c, c \neq a$

15. Ravi and Rashmi are each holding 2 red cards and 2 black cards (all four red and all four black cards are identical). Ravi picks a card at random from Rashmi, and then Rashmi picks a card at random from Ravi. This process is repeated a second time. Let p be the probability that both have all 4 cards of the same colour. Then p satisfies

(A) $p \leq 5\%$ (B) $5\% < p \leq 10\%$ (C) $10\% < p \leq 15\%$ (D) $15\% < p$

Ans. [A]

Sol. If both have all 4 cards of the same color, then there are two possibilities at the end.

Posibility 1 : Ravi holds 4 red cards and Rashmi 4 black cards.

Probability of this possibility = P(Ravi picks red in 1st pick' AND 'Rashmi picks black in 1st pick' AND 'Ravi picks red in 2nd pick' AND 'Rashmi pick black in 2nd pick')

All 4 picks are independent

$= \frac{2}{4} \times \frac{2}{5} \times \frac{1}{4} \times \frac{1}{5} = \frac{1}{100}$

Posibility 2 : Ravi holds 4 black cards and Rashmi 4 red cards similarly probability $= \frac{1}{100}$

$\Rightarrow p = \frac{1}{50} = 0.02 = 2\%$

16. Let A_1, A_2 and A_3 be the regions on R^2 defined by

$A_1 = \{(x, y) : x \geq 0, y \geq 0, 2x + 2y - x^2 - y^2 > 1 > x + y\}$,

$A_2 = \{(x, y) : x \geq 0, y \geq 0, x + y > 1 > x^2 + y^2\}$,

$A_3 = \{(x, y) : x \geq 0, y \geq 0, x + y > 1 > x^3 + y^3\}$,

Denote by $|A_1|$, $|A_2|$ and $|A_3|$ the areas of the regions A_1, A_2, and A_3 respectively. Then

(A) $|A_1| > |A_2| > |A_3|$ (B) $|A_1| > |A_3| > |A_2|$

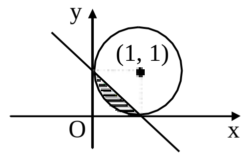
(C) $|A_1| = |A_2| < |A_3|$ (D) $|A_1| = |A_3| > |A_2|$

Ans. **[C]**

Sol. $A_1 = \{(x, y) : x \geq 0, y \geq 0, 2x + 2y - x^2 - y^2 > 1 > x + y\}$

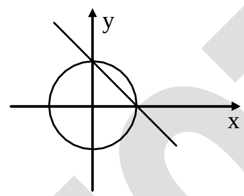
$x^2 + y^2 - 2x - 2y + 1 < 0, x + y < 1$

$(x - 1)^2 + (y - 1)^2 - 1 < 0, x + y < 1$



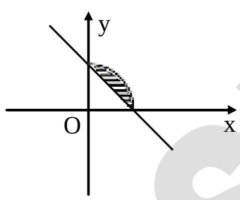
$A_1 = \frac{\pi}{4} - \frac{1}{2}$

$A_2 : x^2 + y^2 < 1 , x + y > 1$



$A_2 = \frac{\pi}{4} - \frac{1}{2}$

A_3 :



$A_3 > A_1 = A_2$

17. Let $f : R \to R$ be a continuous function such that $f(x^2) = f(x^3)$ for all $x \in R$. Consider the following statements.

I. f is an odd function

II. f is an even function

III. f is differentiable everywhere.

Then

(A) I is true and III is false

(B) II is true and III is false

(C) both I and III are true

(D) both II and III are true

Ans. **[D]**

Sol. $f : R \to R$ be a continuous function

Such that

$f(x^2) = f(x^3)$ ..... (i) for all $x \in R$

Put $x = -x$

$f(x^2) = f(-x^3)$

from (i) $f(x^3) = f(-x^3)$

$x^3 = t$ $f(t) = f(-t)$

$\Rightarrow$ f(x) is an even function

(ii) Now take $x^3 = t$

$\Rightarrow f(t^{2/3}) = f(t)$

Put $t = t^{2/3} \Rightarrow f\left(t^{(2/3)^2}\right) = f\left(t^{2/3}\right)$

....

$\Rightarrow f(t) = f\left(t^{2/3}\right) = f\left(t^{(2/3)^2}\right) = f\left(t^{(2/3)^3}\right) .... = f\left(t^{(2/3)^n}\right)$

This is true for all $t \in R$ and any $n \in I$

hence if take $n \to \infty$, $\left(\frac{2}{3}\right)^n \to 0$

Then $f(t) = f(t^0) = 1$

$\Rightarrow$ f(x) is a constant function, hence it is differentiable everywhere

18. Suppose a continuous function $f : [0, \infty) \to R$ satisfies

$f(x) = 2\int_0^x t f(t)dt + 1$ for all $x \geq 0$. Then $f(1)$ equals

(A) e (B) e^2 (C) e^4 (D) e^6

Ans. **[A]**

Sol. $f(x) = 2\int_0^x t\, f(t)dt + 1$, $f(0) = 1$

$f(0) = 1$

$f'(x) = 2x\, f(x)$

$\ln|f(x)| = x^2 + c$

$f(0) = 1, c = 0$

$f(x) = e^{x^2}$

$f(1) = e$

19. Let $a > 0$, $a \neq 1$. Then the set S of all positive real numbers b satisfying $(1 + a^2)(1+ b^2) = 4ab$ is

(A) an empty set (B) a singleton set

(C) a finite set containing more than one element (D) $(0, \infty)$

Ans. **[A]**

Sol. $a > 0, a \neq 1, b \in R^+$

$(1 + a^2)(1 + b^2) = 4ab$

$(1 + a^2)b^2 - 4ab + (1 + a^2) = 0$

$$b = \frac{4a \pm \sqrt{16a^2 - 4(1 + a^2)^2}}{2(1 + a^2)}$$

$$b = \frac{4a \pm 2\sqrt{4a^2 - 1 - a^4 - 2a^2}}{2(1 + a^2)}$$

$$b = \frac{2a \pm (a^2 - 1)i}{1 + a^2}, \quad a \neq 1, a > 0$$

$b \in$ Imaginary

20. Let $f : R \to R$ be a function defined by $f(x) = \begin{cases} \frac{\sin(x^2)}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$. Then, at x = 0, f is

(A) not continuous

(B) continuous but not differentiable

(C) differentiable and the derivative is not continuous

(D) differentiable and the derivative is continuous

Ans. **[D]**

Sol. $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \frac{\sin x^2}{x^2} = \lim\limits_{x\to 0}\left(\frac{\sin x^2}{x^2}\right)x = (1)(0) = 0 = f(0)$

f(x) is continuous

Differentiability :

R.H.D

$$\lim_{h\to 0}\frac{f(0+h)-0}{h} = \lim_{h\to 0}\frac{\frac{\sin h^2}{h}-0}{h} = 1$$

L.H.D.

$$\lim_{h\to 0}\frac{f(0-h)-0}{h} = \lim_{h\to 0}\frac{\frac{\sin h^2}{-h}-0}{-h} = 1$$

R.H.D. = L.H.D.

f(x) is differentiable

$$f'(x) = \frac{x\cos(x^2)2x - \sin(x^2)}{x^2}$$

$$f'(x) = 2\cos x^2 - \frac{\sin x^2}{x^2}$$

$$f'(x) = \begin{cases} 2\cos x^2 - \frac{\sin x^2}{x^2} & , x \neq 0 \\ 1 & , x = 0 \end{cases}$$

Continuity :

$$\lim_{x\to 0} f'(x) = \lim_{x\to 0} 2\cos x^2 - \frac{\sin x^2}{x^2} = 2 - 1 = 1$$

f '(x) is continuous

PHYSICS

21. In a muonic atom, a moun of mass of 200 times of that of electron and same charge is bound to the proton. The wavelengths of its Balmer series are in the range of -

(A) X-rays (B) infrared (C) γ rays (D) microwave

Ans. **[A]**

Sol. $E = \dfrac{-mz^2e^4}{8\epsilon_0^2 h^2 n^2}$

mass = 200 times $\therefore$ energy = 200 times

Hence wavelength decreases.

22. We consider the Thomsan model of the hydrogen atom in which the proton charge is distributed uniformly over a spherical volume of radius 0.25 anstrrom. Applying the Bohr condition in this model the ground state energy (in eV) of the electron will be close to -

(A) $\dfrac{-13.6}{4}$ (B) –13.6 (C) $-\dfrac{13.6}{2}$ (D) -2×13.6

Ans. **[B]**

Sol. $r = 0.529 \dfrac{n^2}{z}$ Å

$r_H = 0.529$ Å

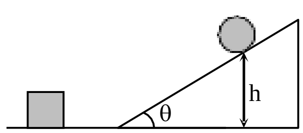
in given Que. r = 0.25Å

which is less than 0.529Å

So, In this case $\boxed{\text{K.E} = \text{P.E.}}$ of e^-

and new T.E. is same as Bohr's model = –13.6 eV

23. A spherical rigid ball is released from rest and starts rolling down and inclined plane from height h = 7 m, as shown in the figure. It hits a block at rest on the horizontal plane (assume elastic collision). If the mass of both the ball and the block is m and the ball is rolling without sliding, then the speed of the block after collision is close to –



(A) 6 m/s (B) 8 m/s (C) 10 m/s (D) 12 m/s

Ans. **[C]**

Sol. $\dfrac{1}{2}mV^2 + \dfrac{1}{2}I\omega^2 = mgh$

$$\frac{1}{2}mV^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\frac{V^2}{R^2} = mgh$$

$$V = \sqrt{\frac{10gh}{7}} = 10 \text{ m/s}$$

24. A girl drops an apple from the window of a train which is moving on a straight track with speed increasing with a constant rate. The trajectory of the falling apple as seen by the girls is -

(A) parabolic and in the direction of the moving train

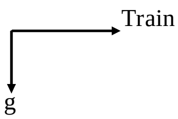
(B) parabolic and opposite to the direction of the moving train

(C) an inclined straight line pointing in the direction of the moving train

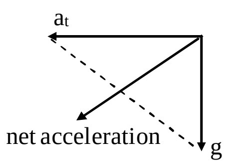
(D) an inclined straight line pointing opposite to the direction of the moving train

Ans. **[D]**

Sol.



Pseudo force is opposite to motion of train



25. A train is moving slowly at 2 m/s next to a railway platform. A man, 1.5 m tall, alights from the train such that his feet are fixed on the ground. Taking him to be a rigid body, the instantaneous angular velocity (in rad/sec) is -

(A) 1.5 (B) 2.0 (C) 2.5 (D) 3.0

Ans. [B]

Sol. $mv\frac{\ell}{2} = I\omega$ $\qquad I = \frac{m\ell^2}{3}$

put the value

$\omega = \frac{3\ell}{2\ell} = 2$ rad/sec.

26. A point mass M moving with a certain velocity collides with a stationary point mass M/2. The collision is elastic and in one dimension. Let the ratio of the final velocity of M and M/2 be x. The value of x is.

(A) 2 (B) 3 (C) 1/2 (D) 1/4

Ans. **[D]**

Sol. $V_1 = \left(\frac{M - \frac{M}{2}}{M + \frac{M}{2}}\right)u_1 = \frac{u^1}{3}$

Similarly $V_2 = \frac{4u_1}{3}$

$\frac{V_1}{V_2} = \frac{1}{4}$

27. A particle of mass 2/3 kg with velocity v = –15 m/s at t = –2 s is acted upon by a force f = k – βt^2. Here k = 8 N and β = 2 N/s^2. The motion is one dimensional. Then the speed at which the particle acceleration is zero again, is -

(A) 1 m/s (B) 16 m/s (C) 17 m/s (D) 32 m/s

Ans. **[C]**

Sol. $mQ = 8 - 2t^2$

$m\dfrac{dV}{dt} = 8 - 2t^2$

$\therefore\ mv = 8t - \dfrac{2t^3}{3} + C$

put the values $C = \dfrac{2}{3}$

$F = 0 \quad \therefore\ ma = 0$ at $t = 2$ sec

$\therefore$ Velocity $v = 17$ m/s

28. A certain stellar body has radius 50 R_s and temperature $2T_s$ and is at a distance of 2×10^{10} A.U. from the earth. Here A.U. refers to the earth sun distance and R_s and T_s refer to the sun's radius and temperature respectively. Take both star and sun to be ideal black bodies. The ratio of the power received on earth from the stellar body as compared to that received from the sun is close to-

(A) 4×10^{-20} (B) 2×10^{-6} (C) 10^{-8} (D) 10^{-16}

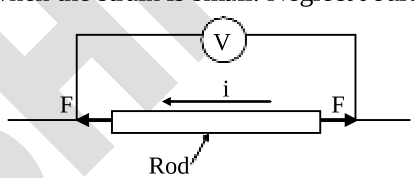
Ans. [D]

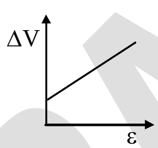
Sol. $P_{Body} = \sigma[4\pi(50R_S)^2]\,(2T)^4$

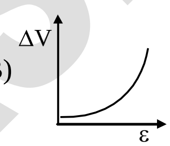
$P_{Body} = 50^2 \times 2^4\, P_{sun}$

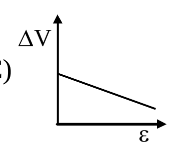
$P_{Body} = 10^{-16}\, I_{Sun}$

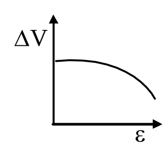
29. As shown in the schematic below, a rod of uniform cross-sectional area A and length l is carrying a constant current i through it and voltage across the rod is measured using an ideal voltmeter. The rod is stretched by the application of a force F. Which of the following graphs would show the variation in the voltage across the rod as function of the strain, ε, when the strain is small. Neglect Joule heating.



(A)


(B)


(C)


(D)


Ans. **[A]**

Sol. $V = iR$

i = constant

$R = \dfrac{\rho\ell}{A}$

$\dfrac{\Delta R}{R} = \left(\dfrac{\Delta\ell}{\ell} - \dfrac{\Delta A}{A}\right)\rho$

$\because\ \rho$ = constant

$\dfrac{\Delta A}{A} = -\dfrac{\Delta\ell}{\ell}$

$\therefore\ \Delta R = 2\rho RE$

$V = i(R_0 + 2\rho\, RE)$

$\therefore$ graph will be [A]

30. Two identical coherent sound sources R and S with frequency f are 5m apart. An observer standing equidistant from the sources and at a perpendicular distance of 12 m from the line RS hears maximum sound intensity when he moves parallel to RS. The sound intensity varies and is a minimum when he comes directly in front of one of the two sources. Then a possible value of f is close to (the speed of sound is 330 m/s)

(A) 495 Hz (B) 275 Hz (C) 660 Hz (D) 330 Hz

Ans. [A]

Sol. $\phi = 25$

$D = 12$

$\Delta r = \frac{d^{\ 2}}{2D}$

$\Delta\phi = \Delta r_p \left(\frac{2\pi}{\lambda}\right)$ $\Delta\phi = (2n + 1)\pi$

$\therefore$ f = 475.2 Hz, n = 1

$f \approx 495$ Hz

31. A photon falls through a height of 1 km through the earth's gravitational field. To calculate the change in its frequency, take its mass to be $h\nu/c^2$. The fractional change in frequency ν is close to -

(A) 10^{-20} (B) 10^{-17} (C) 10^{-13} (D) 10^{-10}

Ans. **[C]**

Sol. $h\nu' = h\nu + mgh$

$\frac{\nu' - \nu}{\nu} = \frac{gh}{c^2} \quad \because m = \frac{h\nu}{c^2}$

$= 1.1 \times 10^{-13}$

32. 0.02 moles of an ideal diatomic gas with initial temperature 20 °C is compressed from 1500 cm^3 to 500 cm^3. The thermodynamic process is such that $PV^2 = \beta$ where β is a constant. Then the value of β is close to. (The gas constant, R = 8.31 J/K/mol)

(A) 7.5×10^{-2} Pa.m^6 (B) 1.5×10^{2} Pa.m^6 (C) 3×10^{-2} Pa.m^6 (D) 2.2×10^{1} Pa.m^6

Ans. **[A]**

Sol. $PV^2 = \beta$

$\because PV = nRT$

$\beta = nRTV = 0.073$

33. A heater supplying constant power P watts is switched on at time t = 0 minutes to raise the temperature of a liquid kept in a calorimeter of negligible heat capacity. A student records the temperature of the liquid T(t) at equal time intervals. A graph is plotted with T(t) on the y-axis versus t on the x-axis. Assume that there is no heat loss to the surrounding heating. Then,

(A) the graph is a straight line parallel to the time axis

(B) the heat capacity of the liquid is inversely proportional to the slope of the graph

(C) if some heat were lost at a constant rate to the surroundings during heating, the graph would be a straight line but with a larger slope

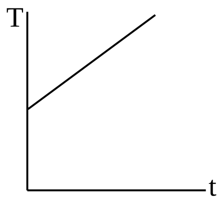
(D) the internal energy of the liquid increases quadratic ally with time

Ans. [B]

Sol. $P = ms\frac{d\theta}{dt}$

$d\theta = \frac{P}{ms}dt$

$\theta = \frac{Pt}{ms} + \theta_0$



34. Unpolarized red light is incident on the surface of a lake at incident angle θ_R. An observer seeing the light reflected from the water surface through a polarizer notices that on rotating the polarizer, the intensity of light drops to zero at a certain orientation. The red light is replaced by unpolarized blue light. The observer sees the same with reflected blue light at incident angle θ_B. then-

(A) $\theta_B < \theta_R < 45°$ (B) $\theta_B = \theta_R$ (C) $\theta_B > \theta_R > 45°$ (D) $\theta_B > \theta_R > 45°$

Ans. [C]

Sol. By Cauchy Theorem

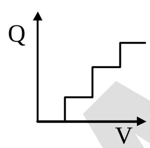
$\mu_{red} < \mu_{Blue}$

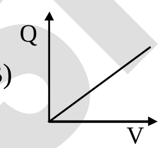
By Brewster law

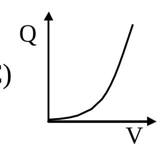
$\mu = \tan i$

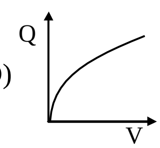
$\therefore \theta_R < \theta_{Blue}$

35. A neutral spherical copper particle has a radius of 10 nm (1 nm = 10^{-9} m). It gets charged by applying the voltage slowly adding one electron at a time. Then the graph of the total charge on the particle vs the applied voltage would look like -

(A) 

(B) 

(C) 

(D) 

Ans. **[A]**

Sol. $V = \frac{kQ}{r}$

and Q = CV

Since e^- is added i.e. at a time therefore Q increases discrete manner

36. A charge +q is distributed over a thin ring of radius r with line charge density $\lambda = q\sin^2\theta/(\pi r)$. Note that the ring is in the x-y plane and θ is the angle made by $\vec{r}$ with the x-axis. The work done by the electric force in displacing a point charge +Q from the centre of the ring to infinity is -

(A) equal to $qQ/2\pi\varepsilon_0 r$

(B) equal to $qQ/4\pi\varepsilon_0 r$

(C) equal to zero only if the path is a straight line perpendicular to the plane of the ring

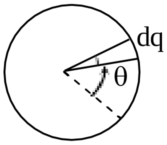
(D) equal to $qQ/8\pi\varepsilon_0 r$

Ans. **[B]**

Sol. $V = \int \frac{kdq}{r}$

Let element one ring has charge dq

$dq = (rd\theta)\lambda$



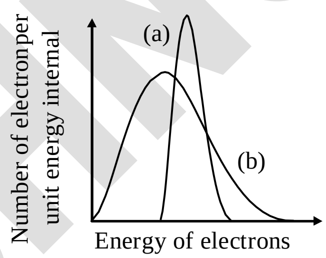
$\therefore\ V = k\int \lambda d\theta$ put the value of λ

$= kq\int_0^{2\pi} \sin^2\left(\frac{\pi}{\pi r}\right) d\theta$

$V = \frac{kq}{r}$

$\therefore\ U = V = \frac{kqQ}{r}$

37. Originally the radioactive beta decay was thought as a decay of a nucleus with the emission of electrons only (Case I). However, in addition to the electron, another (nearly) massless and electrically neutral particle is also emitted (Case II). Based on the figure below, which of the following is correct –



(A) (a) in both cases I and II
(B) (a) in case I and (b) in case II
(C) (a) in case II and (b) in case I
(D) (b) in both cases I and II

Ans. **[B]**

Sol. In case-I, no neutrino, or antineutrino is coming out so energy of β-particle will be same for all the decays.
In case-II, since neutrino or anti-neutrino is also coming out so energy of β-particle will become variable.

38. One gram mole of and ideal gas A with the ratio of constant pressure and constant volume specific heats, $\gamma_A = 5/3$ is mixed with n gram-moles of another ideal gas B with $\gamma_B = 7/5$. If the γ for the mixture is 19/13 what will be the value of n?

(A) 0.75 (B) 2 (C) 1 (D) 3

Ans. **[B]**

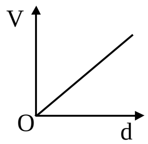
Sol. By law of mixture $f_{min} = \frac{n_1f_1 + n_2f_2}{n_1 + n_2}$

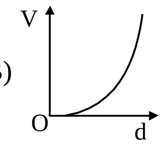
and $\gamma = 1 + \frac{2}{f}$

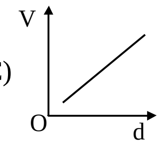
put the values

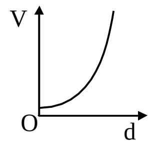
$n_2 = 2$

39. How will the voltage (V) between the two plates of a parallel plate capacitor depend on the distance (d) between the plates, if the charge on the capacitor remains the same ?

(A) 

(B) 

(C) 

(D) 

Ans. **[C]**

Sol. $Q = \frac{\epsilon_0 A}{d} V$

$V = \frac{Qd}{A\,\epsilon_0}$

$\boxed{V \propto d}$

If d = 0, it will not the capacitor there d can't be zero

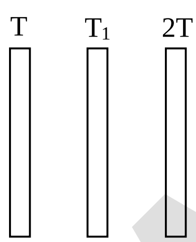
$\therefore$ C will be correct

40. Three large identical plates are kept close and parallel to each other. The outer two plates are maintained at temperatures T and 2T, respectively. The temperature of the middle plate in steady state will be close to -

(A) 1.1 T (B) 1.3 T (C) 1.7 T (D) 1.9 T

Ans. **[C]**

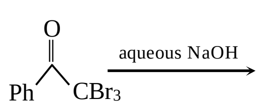
Sol.

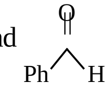


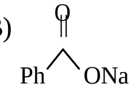
In steady state Heat gain = Heat loss

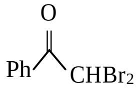
$\therefore 2T_1^4 = T^4 + 16T^4 = 17T^4$

$T_1 = 1.7$ T

41. The major product of the following reaction  are -

(A) $Br_3C - OH$ and 

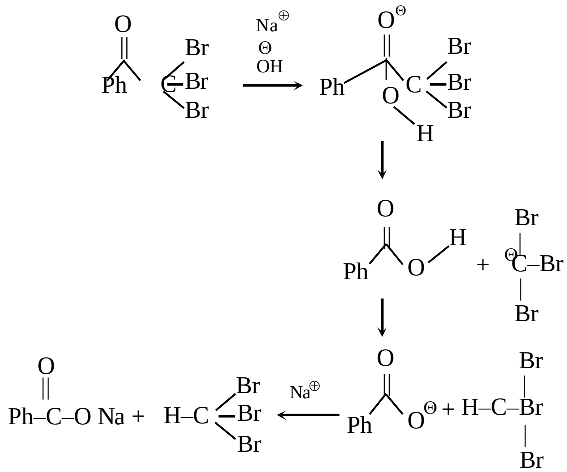
(B)  and $CHBr_3$

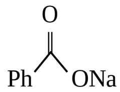
(C)  and NaBr

(D) PhH and CBr_3CO_2Na

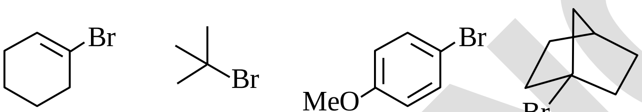
Ans. **[B]**

Sol. Halloform reaction



Option (B)  and $CHBr_3$

42. Among the following,



I II III IV

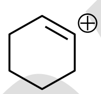
The compounds which can undergo an S_N1 reaction in an aqueous solution, are

(A) I and IV only (B) II and IV only

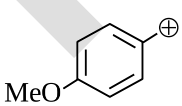
(C) II and III only (D) II, III and IV only

Ans. **[C]**

Sol. Condition for S_N1 reaction is the type of carbocation formed is stable or not

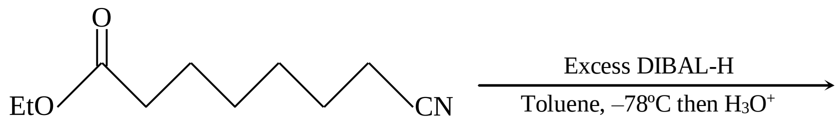
I 

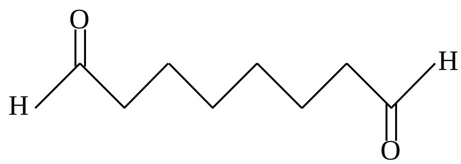
II  Stable through +I effect of three methyl groups

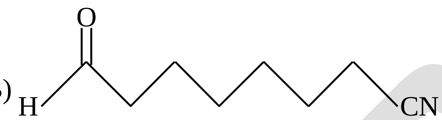
III  Stable due to resonance and delocalization of +ve charge

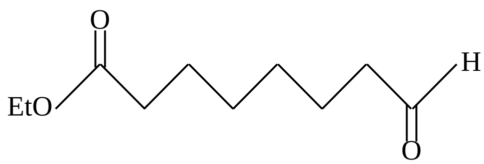
IV Extent of steric hindrance make unstable

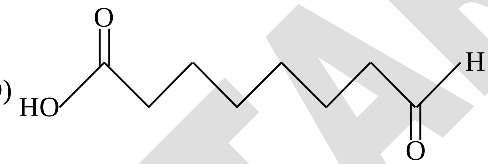
43. The major product of the following reaction

 is

(A) 

(B) 

(C) 

(D) 

Ans. **[A]**

Sol. RCN $\xrightarrow[H_2O]{DIBAL-H}$ R-CHO

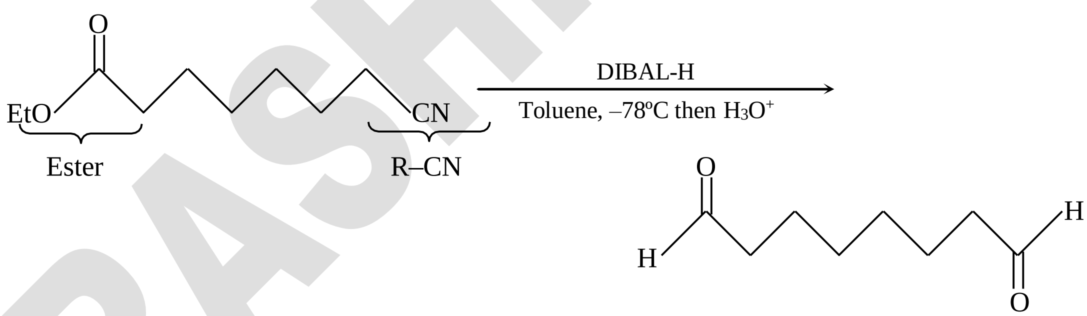
[DIBAL-H] = Al + 1(i-Bu)$_2$

Esters

Example

$$CH_3(CH_2)_9-\overset{\overset{\displaystyle O}{\|}}{C}-OC_2H_5 \xrightarrow[H_2O]{\text{DIBAL-H}} CH_3(CH_2)_9-\overset{\overset{\displaystyle O}{\|}}{C}-H$$

Given -



44. Permanent hardness of water can be removed by

(A) heating

(B) treating with sodium acetate (CH_3CO_2Na)

(C) treating with $Ca(HCO_3)_2$

(D) treatment with sodium hexametaphosphate ($Na_6P_6O_{18}$)

Ans. [D]

Sol. When sodium hexametaphosphate is added to hard water it combines with Ca^{+2}, Mg^{+2} ions present in hard water and forms a complex of calcium and magnesium. These complex don't form any precipitate with soap and hence readily produce lather.

$Na_2[Na_4(PO_3)_6] + 2Ca^{+2}/Mg^{+2} \longrightarrow Na_2[Ca_2/Mg_2\,(PO_3)_6] + 4Na^+$

45. Alkali metals (M) dissolve in liquid NH_3 to give

(A) MNH_2 (B) MH

(C) $[M(NH_3)_x]^+ + [e(NH_3)_y]^-$ (D) M_3N

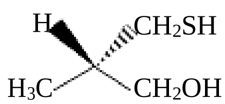
Ans. **[C]**

Sol. Alkali metals (m) dissolve in liquid NH_3 to give

$$M + (x + y)\,NH_3 \longrightarrow \underset{(\text{Ammoniated cation})}{[M(NH_3)_x]^+} + \underset{(\text{Ammoniated anion})}{[e(NH_3)_y]^-}$$

All alkali metals like lithium, sodium, potassium etc dissolves in liquid ammonia to give deep blue coloured solution.

46. The absolute configurations of the following compounds

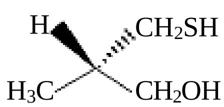


Respectively are

(A) R and R (B) S and S (C) R and S (D) S and R

Ans. **[D]**

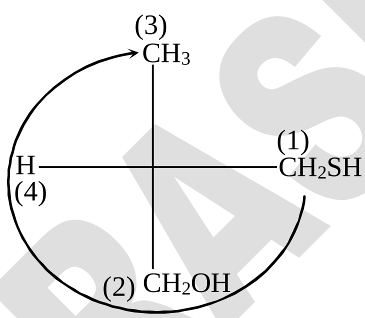
Sol.



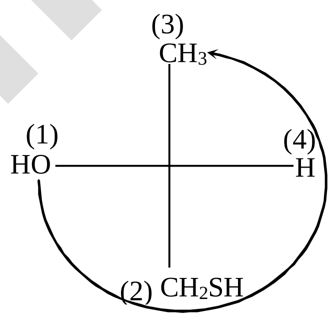
I

II

Firstly draw fischer project i.e. in 2-D, then assign priorities, then check clockwise or anticlockwise



Clockwise



Anticlockwise

So it should be 'R' but here, lower priority group is on horizontal line Therefore configuration will be reversed
i.e. correct configuration 'S'

It should be 'S' lower priority group is on horizontal line
i.e. 'R'

47. The diamagnetic species among the following is

(A) O_2^+ (B) O_2^- (C) O_2 (D) O_2^{2-}

Ans. **[D]**

Sol. Diamagnetic species

(A) O_2^{+} (15e$^-$) = $\sigma1s^2\sigma^*1s^2\sigma2s^2\sigma^*2s^2\sigma2p_z^{\,2}(\pi2p_x^{\,2} = \pi2p_y^{\,2})\ (\pi^*2p_x^{\,1} = \pi^*2p_y)$

Number of up e$^-$ = 1 $\Rightarrow$ Paramagnetic

(B) $O_2^{\,-}$ = (17e$^-$) = $\sigma1s^2\sigma^*1s^2\sigma2s^2\sigma^*2s^2\sigma2p_z^{\,2}(\pi2p_x^{\,2} = \pi2p_y^{\,2})\ (\pi^*2p_x^{\,2} = \pi^*p_y^{\,1})$

Number of up e$^-$ = 1 $\Rightarrow$ Paramagnetic

(C) O_2 (16 e$^-$) = $\sigma1s^2\sigma^*1s^2\sigma2s^2\sigma^*2s^2\sigma2p_z^{\,2}(\pi2p_x^{\,2} = \pi2p_y^{\,2})\ (\pi^*2p_x^{\,2} = \pi^*p_y^{\,1})$

Number of up e$^-$ = 2 $\Rightarrow$ Paramagnetic

(D) $O_2^{\,2-}$ (18 e$^-$) = $\sigma1s^2\sigma^*1s^2\sigma2s^2\sigma^*2s^2\sigma2p_z^{\,2}(\pi2p_x^{\,2} = \pi2p_y^{\,2})\ (\pi^*2p_x^{\,2} = \pi^*p_y^{\,2})$

Number of up e$^-$ = 2 $\Rightarrow$ diamagnetic

48. Among the following transformations, the hybridization of the central atom remains unchanged in -

(A) $CO_2 \longrightarrow HCOOH$ (B) $BF_3 \longrightarrow BF_4^-$

(C) $NH_3 \longrightarrow NH_4^+$ (D) $PCl_3 \longrightarrow PCl_5$

Ans. **[C]**

Sol. (A) $CO_2 \longrightarrow HCOOH$

sp $\quad$ O = C – O – H $\Rightarrow$ sp^2 (with H on C)

(B) $BF_3 \longrightarrow BF_4^-$

½ (3 + 3) = 3 $\quad$ ½ (3 + 4 + 1)

sp^2 $\quad$ sp^3

(C) $NH_3 \longrightarrow NH_4^+$

½ (5 + 3) = 4 $\quad$ ½ (5 + 4 – 1) = 4

Sp^3 $\quad$ sp^3

(D) $PCl_3 \longrightarrow PCl_5$

½ (5 + 3) = 4 $\quad$ ½ (5 + 5) = 5

Sp^3 $\quad$ sp^{3}d

49. For an octahedral complex MX_4Y_2 (M = a transition metal, X and Y are monodentate achiral ligands), the correct statement, among the following, is

(A) MX_4Y_2 has 2 geometrical isomers one of which is chiral

(B) MX_4Y_2 has 2 geometrical isomers both of which are chiral

(C) MX_4Y_2 has 4 geometrical isomers all of which are chiral

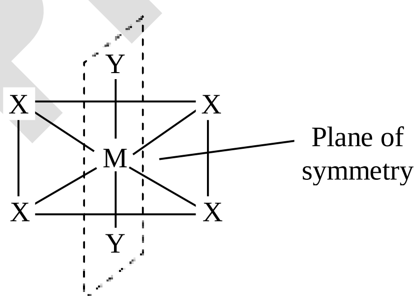
(D) MX_4Y_2 has 4 geometrical isomers two of which are chiral

Ans. **[B]**

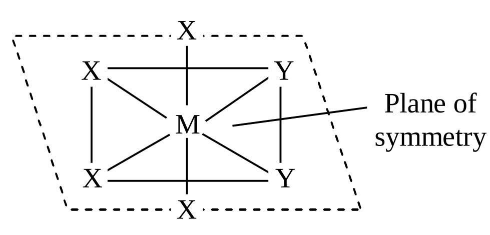
Sol. $MX_4Y_2 \longrightarrow$ Octahedral complex

Key point – If their is plane of symmetry in a complex, then it will be achiral and optically inactive

Geometrical isomers means exist in cis and trans forms



trans
(Achiral)



Cis
(Achiral)

i.e. two geometrical isomers both of which are achiral

50. The values of the Henry's law constant of Ar, CO_2, CH_4 and O_2 in water at 25ºC are 40.30, 1.67, 0.41 and 34.86 kbar, respectively. The order of their solubility in water at the same temperature and pressure is

(A) $Ar > O_2 > CO_2 > CH_4$ (B) $CH_4 > CO_2 > Ar > O_2$

(C) $CH_4 > CO_2 > O_2 > Ar$ (D) $Ar > CH_4 > O_2 > CO_2$

Ans. **[C]**

Sol. Henry's law $P = K_H X_{gas}$

k_H – Henrys constant

P = parital pressure of gas above liquid surface

Solubility of gas in liquied $\propto$ parital pressure of gas above liquied surface

$k_H \uparrow \Rightarrow P \downarrow \Rightarrow$ Solubility $\downarrow$

$k_H \Rightarrow Ar > O_2 > CO_2 > CH_4$

4.30 34.86 1.67 0.41

Solubility $\Rightarrow Ar < O_2 < CO_2 < CH_4$

51. Thermal decomposition of N_2O_5 occurs as per the equation below

$2N_2O_5 \longrightarrow 4\,NO_2 + O_2$

The correct statement is

(A) O_2 production rate is four times the NO_2 production rate

(B) O_2 production rate is the same as the rate of disappearance of N_2O_5

(C) rate of disappearance of N_2O_5 is one-fourth of NO_2 production rate

(D) rate of disappearance of N_2O_5 is twice the O_2 production rate

Ans. **[D]**

Sol. $2N_2O_5 \longrightarrow 4\,NO_2 + O_2$

$$\frac{-d[N_2O_5]}{2dt} = \frac{d[NO_2]}{4dt} = \frac{d[O_2]}{dt}$$

Production Rate of O_2 = $\frac{d[O_2]}{dt}$

Production Rate of NO_2 = $\frac{d[NO_2]}{dt}$

Disappearance Rate of N_2O_5 = $\frac{-d[N_2O_5]}{dt}$

$$\frac{-d[N_2O_5]}{dt} = \frac{2d[O_2]}{dt}$$

[Rate of disappearance of N_2O_5 = Twice of production of O_2]

52. For a 1st order chemical reaction.

(A) the product formation rate is independent of reactant concentration

(B) the time taken for the completion of half of the reaction ($t_{1/2}$) is 69.3% of the rate constant (k)

(C) the dimension of Arrhenius pre-exponential factor is reciprocal of time

(D) the concentration vs time plot for the reactant should be linear with a negative slope

Ans. **[C]**

Sol. For 1st order chemical reaction

$$t = \frac{2.303}{k}\log_{10}\frac{[A_0]}{[A_t]}$$

$$t_{1/2} = \frac{0.693}{k}$$

Arrhenius equation : $\boxed{K = A.e^{-E_a/RT}}$

[A = Pre-exponential factor]

In a first order reaction the units of the pre-exponential factor are reciprocal seconds. Because the pre-exponential factor depends on frequency of callisions. Its releated to collision theory and trarution state theory

53. The boiling point of 0.001 M aqueous solutions of NaCl, Na_2SO_4, K_3PO_4 and CH_3COOH should follows the order

(A) $CH_3COOH < NaCl < Na_2SO_4 < K_3PO_4$

(B) $NaCl < Na_2SO_4 < K_3PO_4 < CH_3COOH$

(C) $CH_3COOH < K_3PO_4 < Na_2SO_4 < NaCl$

(D) $CH_3COOH < K_3PO_4 < NaCl < Na_2SO_4$

Ans. **[A]**

Sol. C = 0.001 m

NaCl, Na_2SO_4, K_3PO_4, CH_3COOH

[$\Delta T_b = K_b \times m \times i$]

Boiling point depends on the malality (m) & Van't Haff Factor (i)

$m\uparrow$ & $i\uparrow \Rightarrow$ Boiling Point $\uparrow$

For NaCl $\Rightarrow$ Strong electrolyte i = 2

For $Na_2SO_4 \Rightarrow$ Strong electrolyte i = 3

For $K_3PO_4 \Rightarrow$ Strong electrolyte i = 4

CH_3COOH is WEAK ACID

Boiling point : $K_3PO_4 > Na_2SO_4 > NaCl > CH_3COOH$

Option (A)

54. An allotrope of carbon which exhibits only two types of C–C bond distance of 143.5 pm and 138.3 pm, is -

(A) charcoal (B) graphite (C) diamond (D) fullerence

Ans. **[D]**

Sol. Fact – Fullerene a soccer ball shaped molecule has 60 vertices with a carbon atom at each vertex. It contains both single and double bond with C–C at a distance of 143.5 pm and 138.3 pm

Whereas in diamond C–C bond length – 154 pm and in graphite 140 pm.

55. Nylon-2-nylon-6 is a co-polymer of 6-aminohexanoic acid and

(A) glycine (B) valine (C) alanine (D) leucine

Ans. **[A]**

Sol. Nylon-2-nylon-6 is a copolymer of –

Glycine and Amino caproic acid

(fact)

$$\underset{\text{Glycine}}{nH_2N{-}CH_2{-}COOH} + \underset{\text{Aminocaproic acid}}{nNH_2{-}(CH_2)_5{-}COOH} \xrightarrow{-\,H_2O} \left[\underset{\underset{O}{\|}}{C}{-}NH_2{-}NH{-}\underset{\underset{O}{\|}}{C}{-}(CH_2)_5\,NH \right]$$

Nylon-2-nylon-6

56. A solid is hard and brittle. It is an insulator is solid state but conducts electricity in molten state. The solid is a

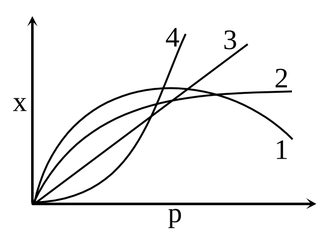
(A) molecular solid (B) ionic solid (C) metallic solid (D) covalent solid

Ans. **[B]**

Sol. Ionic solid is hard and brittle. Ex. NaCl

It is an insulator in solid state but in molten state it conducts electricity as it has free ions in melten state.

57. The curve that best describes the adsorption of a gas (x g) on 1.0 g of a solid substrate as a function of pressure (p) at a fixed temperature -

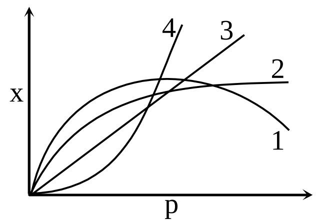


is

(A) 1 (B) 2 (C) 3 (D) 4

Ans. **[B]**

Sol.



According to, Freundlich adsorption isotherm –

$x/m = K.p^{1/n}$ (n > 1)

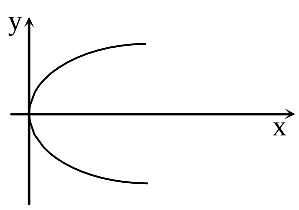
m = 1g (given)

$x = kp^{1/n}$

$\frac{x}{k} = p^{1/n}$

$\left(\frac{x}{k}\right)^n = p$

$y^n = x$ (Equation of parabola). If n = 2



$\Rightarrow$ graph (2) represent this situation

58. The octahedral complex $CoSO_4Cl.5NH_3$ exists in two isomeric forms X and Y. Isomer X reacts with $AgNO_3$ to give a white precipitate, but does not react with $BaCl_2$, Isomer Y gives white precipitate with $BaCl_2$ but does react with $AgNO_3$.

Isomers X and Y are

(A) ionization isomers (B) Linkage isomers

(C) coordination isomers (D) solvate isomers

Ans. **[A]**

Sol. $CoSO_4Cl_5.NH_3$

Isomers
- (I) $[Co(NH_3)_5SO_4]Cl$
- (II) $[Co(NH_3)_5Cl]SO_4$

$\Rightarrow$ Ionization isomers

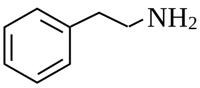
(I) $[Co(NH_3)_5SO_4]Cl + AgNO_3 \longrightarrow AgCl + [Co(NH_3)_5SO_4]NO_3$

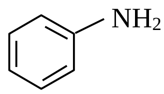
[White precipitate]

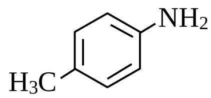
(II) $[Co(NH_3)_5Cl]SO_4 + BaCl_2 \longrightarrow BaSO_4$

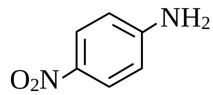
[White precipitate]

59. The correct order of basicity of the following amines


I


II


III


IV

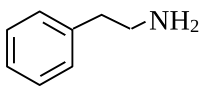
is -

(A) I > II > III> IV (B) I > III > II > IV

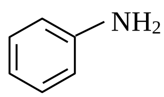
(C) III > II > I > IV (D) IV > III > II > I

Ans. **[B]**

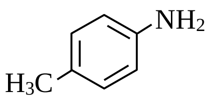
Sol.


I

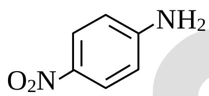
No resonance of Lone pair of "N"


II

No effect resonance of Lone pair of "N"


III

+I of CH_3 and resonance of Lone pair of "N"


IV

–I effect of $–NO_2$

Basic strength : I > III > II > IV

60. Electrolysis of a concentrated aqueous solution of NaCl results in

(A) increase in pH of the solution (B) decrease in pH of the solution

(C) O_2 liberation at the cathode (D) H_2 liberation at the anode

Ans. **[A]**

Sol. Electrolysis of aqueous NaCl solution if known as chlor alkali process (production of caustic soda NaOH)

$NaCl + H_2O \xrightarrow{\textit{electricity}} NaOH + Cl_2 + H_2$

Production of NaOH $\therefore \uparrow$ in pH

$\therefore$ option (A)

H_2 liberation will be at cathode due to ($H^+ \rightarrow H_2$ gain of eldctron at cathode)

$\therefore$ option (D) is wrong

BIOLOGY

61. Ethanol is used to treat methanol toxicity because ethanol

(A) is a competitive inhibitor of alcohol dehydrogenase

(B) is a non-competitive inhibitor of alcohol dehydrogenase

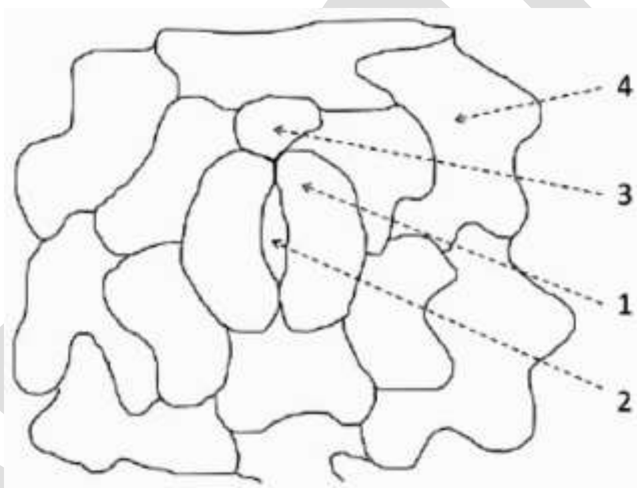
(C) activates enzymes involved in methanol metabolism

(D) inhibits methanol uptake by cells

Ans. [A]

Sol. Alcohol dehydrogenes facilitate the conversion of methaneol to formaldehyde (CH_2O) which is a potent poison. Ethanol acts as competitive inhibitors of alcohol dehydrogenase. Ethanol has a higher binding affinity to alcohol dehydrogenase compared to methanol resulting in blockade of the formation of toxic compounds.

62. Given below is a diagram of stomatal apparatus. Match the labels with the corresponding names of the components.



Choose the CORRECT combination.

(A) 1 – Stomatal pore; 2 – Guard cell; 3 – Epidermal cell; 4 – Subsidiary cell

(B) 1 – Guard cell; 2 – Stomatal pore; 3 – Subsidiary cell; 4 – Epidermal cell

(C) 1 – Subsidiary cell; 2 – Guard cell; 3 – Stomatal pore; 4 – Epidermal cell

(D) 1 – Guard cell; 2 – Stomatal pore; 3 – Epidermal cell; 4 – Subsidiary cell

Ans. **[B]**

Sol. Label markings are as follows

1 – Guard cell

2 – Stomatal pore

3 – Subsidiary cell

4 – Epidermal cell

63. Which one of following pairs was excluded from Whittaker's five kingdom classification?

(A) Viruses and lichens (B) Algae and euglena (C) Lichens and algae (D) Euglena and viruses

Ans. [A]

Sol. The five kingdom classification system consist of monera, protista, fungi, plantae & animalia. Out of the options, Algae is placed in plant kingdom & Euglena is placed in protista. So Virus & lichens are not included in any kingdom.

64. A plant species when grown in shade tends to produce thinner leaves with more surface area, and when grown under abundant sunlight starts producing thicker leaves with reduced surface area. This phenomenon is an example of

(A) character displacement (B) phenotypic plasticity (C) natural selection (D) genotypic variation

Ans. [B]

Sol. Phenotypic plasticity is considered one of the major means by which plants can cope with environmental factor variability.

65. Sacred groves found in several regions in India are an example of

(A) in situ conservation (B) ex situ conservation (C) reintroduction (D) restoration

Ans. **[A]**

Sol. Sacred groves are considered in In-situ conservation because they involve protection of all trees & wild life within them are protected at the site of their location. Ex-Aravalli Hills of Rajasthan, Western Ghat etc.

66. Which one of the following immune processes is most effectively controlled by anti-histamines ?

(A) Cell-mediated autoimmunity

(B) IgE-mediated exaggerated immune response

(C) IgG-mediated humoral immune response

(D) IgM-mediated humoral immune response

Ans. [B]

Sol. When an allergen comes in contact with antibody IgE then IgE binds to the allergen which causes the mast cells to release Histamine. Histamine further increases allergic reactions like coughing, sneezing etc. Anti histamine blocks the synthesis of Histamine.

67. Which one of the following is explained by the endosymbiotic theory ?

(A) The interaction between bacteria and viruses

(B) The symbiosis between plants and animals

(C) The origin of mitochondria and chloroplast

(D) The evolution of multicellular organisms from unicellular ones

Ans. **[C]**

Sol. Endo symbiotic theory deals with the origins of mitochondria and chloroplast.

The mitochondria & chloroplast are eukaryotic organelles that have bacterial characteristics.

68. According to the logistic population growth model, the growth rate is independent of

(A) per capita birth rate (B) per capita death rate (C) resource availability (D) environmental fluctuations

Ans. [**D**]

Sol. Logistic growth $\rightarrow \dfrac{dN}{dt} = rN\left(\dfrac{K-N}{K}\right)$

Here the growth rate is independent of environment fluctuations.

69. A violent volcanic eruption wiped out most of the life forms in an island. Over time, different forms of simple organisms colonised this region, followed by the emergence of other organisms such as shrubs, woody plants, invertebrates and mammals. This ecological process is referred to as

(A) generation (B) replacement (C) succession (D) turnover

Ans. [C]

Sol. Succession is a process that starts in an area where no living organisms are there.

70. Which one of the following microbial product is called "clot buster" ?

(A) Cyclosporin A (B) Paracetamol (C) Statins (D) Streptokinase

Ans. **[D]**

Sol. Streptokinase is used to break-down clots.

71. Which one of the following elements is NOT directly involved in transcription ?

(A) Promoter (B) Terminator (C) Enhancer (D) OriC

Ans. **[D]**

Sol. OriC is origin of replication which does not take part in transcription.

72. Which one of the following phyla is a pseudocoelomate ?

(A) Cnidaria (B) Nematoda (C) Mollusca (D) Chordate

Ans. **[B]**

Sol. Nematoda

73. Which one of the following glands does NOT secrete saliva ?

(A) Submaxillary gland (B) Lacrimal gland (C) Parotid gland (D) Sublingual gland

Ans. **[B]**

Sol. Lacrimal gland secretes the aqueous layer of the tear film.

74. Which one of the following options correctly represents the tissue arrangement in roots ?

(A) Cortex, pericycle, casparian strip, vascular bundle

(B) Pericycle, cortex, casparian strip, vascular bundle

(C) Cortex, casparian strip, pericycle, vascular bundle

(D) Casparian strip, pericycle, cortex, vascular bundle

Ans. **[C]**

Sol. Cortex $\rightarrow$ Casparian strip $\rightarrow$ Pericycle $\rightarrow$ Vascular bundle

75. During fermentation of glucose to ethanol, glucose is

(A) first reduced and then oxidised

(B) only oxidised

(C) neither oxidised nor reduced

(D) only reduced

Ans. **[C]**

Sol. When considering glycolysis & fermentation, there is no oxidation & reduction of glucose.

76. Which of the following is/are the product (s) of cyclic photophosphorylation?

(A) Both NADPH and H^+

(B) NADPH

(C) ATP

(D) Both ATP and NADPH

Ans. **[C]**

Sol. In cyclic photophosphorylation, only PS-I is functional the electron is circulated within the photosystem and the phosphorylation occurs due to cyclic flow of electrons.

The excited electrons does not pass on to $NADP^+$ but is cycled back to the PS-I Complex through ETS. Thus cyclic flow only results in synthesis of ATP but not NADPH + H^+

77. Which one of the following amino acids is least likely to be in the core of a protein?

(A) Phenylalanine (B) Valine (C) Isoleucine (D) Arginine

Ans. **[D]**

Sol. Phenylalanine, Valine & Isoleucine are hydrophobic amino acids which are generally found at the core of protein but Arginine is a charged amino acid which is generally found at surface.

78. Which one of following statements is a general feature of global species diversity ?

(A) It increases from high to low latitudes

(B) It increases from low to high latitudes

(C) It changes over time but not spatially

(D) It changes randomly across space and time

Ans. **[A]**

Sol. Biodiversity increases from high to low latitudes as there is optimum temperature available in low latitudes then high latitudes.

79. Which one of the following conditions is NOT responsible for the presence of deoxygenated blood in the arteries of a newborn ?

(A) Pneumonia

(B) Atrial septal defect

(C) Shunt between pulmonary artery and aorta

(D) Phenylketonuria

Ans. **[D]**

Sol. Phenylketonuria is an in born error of metabolism. The affected individual lacks an enzyme that converts the amino acid phenylalanine into tyrosine. Accumulation of phenyl pyruvic acid results in mental retardation.

80. Rhizobium forms symbiotic association with roots in legumes and fixes atmospheric nitrogen. Which one of the following statement is CORRECT about this process ?

(A) Activity of nitrogenase is sensitive to oxygen

(B) Activity of nitogenase is insensitive to oxygen

(C) Anaerobic conditions allow ATP independent conversion of nitrogen to ammonia

(D) Under aerobic conditions, atmospheric nitrogen can be converted to nitrates by Rhizobium

Ans. **[A]**

Sol. Oxygen inhibits the nitrogenase activity.

Part – II
Two-Mark Questions

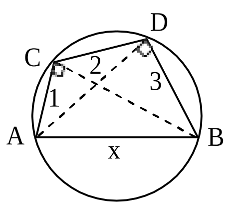
MATHEMATICS

81. The points C and D on a semicircle with AB as diameter are such that AC = 1, CD = 2, and DB = 3. Then the length of AB lies in the interval

(A) [4, 4.1)   (B) [4.1, 4.2)   (C) [4.2, 4.3)   (D) [4.3, ∞)

Ans. **[B]**

Sol.



Let AB = x

By Ptotemy's Theorem

AC × BD + AB × CD = AD × BC

$1\times 3 + x \times 2 = \sqrt{x^2-9}\,\sqrt{x^2-1}$

$9 + 4x^2 + 12x = x^4 - 10x^2 + 9$

$x^4 - 14x^2 - 12x = 0$

$x(x^3 - 14x - 12) = 0 \quad (x \neq 0)$

Take $f(x) = x^3 - 14x - 12$

$f(4) = -4$

$f(4.1) = -0.479$

$f(4.2) = 3.288$

$f(4.3) = 7.307$

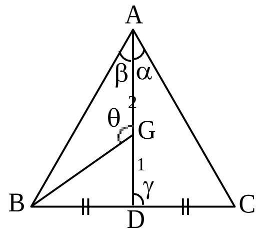
as f(x) is a continuous function, therefore one root of f(x) lies in [4.1, 4.2) , i.e. length of AB lies in this interval

82. Let ABC be a triangle and let D be the midpoint of BC. Suppose cot(∠CAD): cot(∠BAD) = 2 : 1. If G is the centroid of triangle ABC, then the measure of ∠BGA is

(A) 90°   (B) 105°   (C) 120°   (D) 135°

Ans. **[A]**

Sol.



Given

$$\frac{\cot(\angle CAD)}{\cot(\angle BAD)} = \frac{2}{1}$$

$\cot\alpha = 2\cot\beta$ ...(i)

In ΔABC

$$\frac{BD}{DC} = 1:1$$

Apply m-n cot Theorem

$(1 + 1)\cot\gamma = 1\cdot\cot\beta - 1\cdot\cot\alpha$

$2\cot\gamma = -\cot\beta$ ...(ii) In

ΔBAD

$$\frac{DG}{GA} = \frac{1}{2} \quad \{G \text{ is centroid}\}$$

Apply m-n cot Theorem :

$(1 + 2)\cot\theta = 2\cot(\angle ADB) - 1\cdot\cot(\angle BAD)$

$\Rightarrow 3\cot\theta = 2\cot(\pi - \gamma) - \cot\gamma^3$

$\Rightarrow 3\cot\theta = -2\cot\gamma + 2\cot\gamma$

$\Rightarrow \cot\theta = 0$

$\Rightarrow \theta = \frac{\pi}{2}$

83. Let $f(x) = x^6 - 2x^5 + x^3 + x^2 - x - 1$ and $g(x) = x^4 - x^3 - x^2 - 1$ be two polynomials. Let a, b, c, and d be the roots of $g(x) = 0$. Then the value of $f(a) + f(b) + f(c) + f(d)$ is

(A) – 5 (B) 0 (C) 4 (D) 5

Ans. **[B]**

Sol. If a is a root of $g(x) = 0$, then $g(a) = 0$

$a^4 - a^3 - a^2 - 1 = 0$ ....(i)

Now,

$f(a) = a^6 - 2a^5 + a^3 + a^2 - a - 1$

$f(a) = a^2\underbrace{(a^4 - a^3 - a^2 - 1)}_{=0} - a^5 + a^4 + a^3 + 2a^2 - a - 1 = 0$

$f(a) = -a\underbrace{(a^4 - a^3 - a^2 - 1)}_{=0} + 2a^2 - 2a - 1$

$f(a) = 2a^2 - 2a - 1$

Similarly we can write f(b), f(c), f(d)

Now, a, b, c, d are root of $x^4 - x^3 - x^2 - 1 = 0$

$\sum a = 1$

$\sum ab = -1$

$f(a) + f(b) + f(c) + f(d)$

$= 2\sum a^2 - 2\sum a - \sum 1$

$= 2\left[\left(\sum a\right)^2 - 2\sum ab\right] - 2 - 4$

$= 2[1 + 2] - 6 = 0$

84. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$, and $\vec{c} = 5\hat{i} + \hat{j} - \hat{k}$ be three vectors. The area of the region formed by the set of points whose position vectors r satisfy the equations r.a = 5 and $|\vec{r} - \vec{b}| + |\vec{r} - \vec{c}| = 4$ is closest to the integer

(A) 4 (B) 9 (C) 14 (D) 19

Ans. **[A]**

Sol. (i) $\vec{r} \cdot \vec{a} = 5$

This is an equation of plane

(ii) $|\vec{r} - \vec{b}| + |\vec{r} - \vec{c}| = 4$

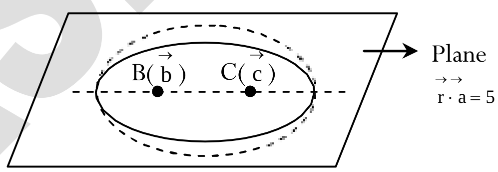
i.e. sum of distances of a point $(\vec{r})$ from two fixed points with position vector $\vec{b}$ and $\vec{c}$ is constant (Also check $|\vec{b} - \vec{c}| = \sqrt{14} < 4$)

$\Rightarrow$ such points lies on ellipsoid (as in 2D, such points lies on ellipse)

Now points with p.v. $\vec{b}$ & $\vec{c}$ satisfies the equation of plane $\vec{r} \cdot \vec{a} = 5$

$\vec{b} \cdot \vec{a} = 5$

$\vec{c} \cdot \vec{a} = 5$



Area in the plane constitutes an ellipse

Distance between $\vec{b}$ and $\vec{c}$ = 2 × (semi major axis) × e = $\sqrt{14}$

$2ae = \sqrt{14}$ (i)

Sum of distance = constant = major axis = 4

2a = 4 ....(ii)

$\Rightarrow e = \dfrac{\sqrt{14}}{4} \Rightarrow b = \dfrac{1}{\sqrt{2}}$ (semi minor axis)

Area of ellipse = $\pi \cdot a \cdot b$

$= \pi \cdot 2 \cdot \dfrac{1}{\sqrt{2}} = \sqrt{2}\pi \approx 4.443$

85. The number of solutions to $\sin(\pi \sin^2(\theta)) + \sin(\pi \cos^2(\theta)) = 2\cos\left(\frac{\pi}{2}\cos(\theta)\right)$ satisfying $0 \le \theta \le 2\pi$ is

(A) 1 (B) 2 (C) 4 (D) 7

Ans. **[D]**

Sol. $\sin(\pi \sin^2\theta) + \sin(\pi \cos^2\theta) = 2\cos\left(\frac{\pi}{2}\cos\theta\right)$

$2\sin\left(\frac{\pi}{2}\right).\cos\left(\frac{\pi}{2}\cos 2\theta\right) = 2\cos\left(\frac{\pi}{2}\cos\theta\right)$

$\cos\left(\frac{\pi}{2}\cos 2\theta\right) = \cos\left(\frac{\pi}{2}\cos\theta\right)$

$\frac{\pi}{2}\cos 2\theta = 2n\pi \pm \frac{\pi}{2}\cos\theta$

take + ve

$\frac{\pi}{2}(\cos 2\theta - \cos\theta) = 2n\pi$

$\cos 2\theta - \cos\theta = 4n,\ n = 0$

$2\cos^2\theta - \cos\theta - 1 = 0$

$\underbrace{\cos\theta = -\frac{1}{2}}_{2 \text{ solution}},\ \underbrace{\cos\theta = 1}_{2 \text{ solution}}$

take – ve

$\frac{\pi}{2}\cos 2\theta = 2n\pi - \frac{\pi}{2}\cos\theta$

$\cos 2\theta + \cos\theta = 4n,\ n = 0$

$2\cos^2\theta + \cos\theta - 1 = 0$

$\underbrace{\cos\theta = \frac{1}{2}}_{\text{Two solution}},\ \underbrace{\cos\theta = -1}_{\text{One solution}}$

total 7 soution in $\theta \in [0, 2\pi]$,

86. Let $J = \int_0^1 \frac{x}{1 + x^8}dx$. Consider the following assertions:

I. $J > \frac{1}{4}$

II. $J < \frac{\pi}{8}$

Then

(A) only I is true (B) only II is true

(C) both I and II are true (D) neither I nor II is true

Ans. **[A]**

Sol. $J = \int_0^1 \frac{x}{1+x^8}dx$

$\because 0 < x^8 < 1$

$J > \int_0^1 \frac{x}{2}dx$

$J > \frac{1}{2}\cdot\left.\frac{x^2}{2}\right|_0^1$

$J > \frac{1}{4}$

$J < \int_0^1 \frac{x}{1+0}dx$

$J < \left.\frac{x^2}{2}\right|_0^1$

$J < \frac{1}{2}$

(I) is true. II is false

87. Let $f : (-1, 1) \to R$ be a differentiable function satisfying $(f'(x))^4 = 16(f(x))^2$ for all $x \in (-1, 1)$, $f(0) = 0$. The number of such function is

(A) 2 (B) 3 (C) 4 (D) more than 4

Ans. **[D]**

Sol. Given $(f'(x))^4 = 16\,(f(x))^2$ for all $x \in (-1, 1)$

$f(0) = 0$

$\Rightarrow (f'(x))^2 = \pm 4f(x)$

$\Rightarrow f'(x) = \pm 2\sqrt{\pm f(x)}$

(i) **<u>Case 1</u>**

$f'(x) = 2\sqrt{f(x)}$

$\int \frac{d(f(x))}{\sqrt{f(x)}} = \int 2dx \Rightarrow 2\sqrt{f(x)} = 2x + c$

$f(0) = 0 \Rightarrow c = 0$

$\Rightarrow \sqrt{f(x)} = x$

$\Rightarrow x \geq 0$

$f(x) = x^2 \,;\, 0 \leq x < 1$

(ii) **Case 2**

$f'(x) = -2\sqrt{f(x)}$

$\sqrt{f(x)} = -x \Rightarrow x \le 0$

$f(x) = x^2 ; -1 < x \le 0$

(iii) **Case 3**

$f'(x) = 2\sqrt{-f(x)}$

$\sqrt{-f(x)} = x$

$\Rightarrow f(x) = -x^2 ; 0 \le x < 1$

(iv) **Case 4**

$f'(x) = -2\sqrt{-f(x)}$

$\sqrt{-f(x)} = -x$

$\Rightarrow f(x) = -x^2, -1 < x \le 0$

(v) Also, one singular solution of given differential equation is

$f(x) = 0, -1 < x < 1$

Hence, there are more than 4 function possible

$f_1(x) = \begin{cases} x^2; & 0 \le x < 1 \\ -x^2; & -1 < x < 0 \end{cases}$ $\quad f_2(x) = \begin{cases} -x^2; & 0 \le x < 1 \\ x^2; & -1 < x < 0 \end{cases}$

$f_3(x) = x^2 ; -1 < x < 1$ $\quad f_4(x) = -x^2 ; -1 < x < 1$

$f_5(x) = 0 ; -1 < x < 1$ ....

88. For $x \in R$, let $f(x) = |\sin x|$ and $g(x) = \int_0^x f(t)dt$. Let $p(x) = g(x) - \frac{2}{\pi}x$. Then

(A) $p(x + \pi) = p(x)$ for all x

(B) $p(x + \pi) \ne p(x)$ for at least one but finitely many x

(C) $p(x + \pi) \ne (x)$ for infinitely many x

(D) p is a one-one function

Ans. **[A]**

Sol. Given $f(x) = |\sin x|$

$g(x) = \int_0^x f(t) \cdot dt$

$p(x) = g(x) - \frac{2}{\pi}x$

Now, $p(x + \pi) = g(x + \pi) - \frac{2}{\pi}(x + \pi)$

$= \int_0^{\pi+x} f(t)dt. - \frac{2}{\pi}x - 2$

$= \int_0^{\pi} f(t)dt + \int_{\pi}^{\pi+x} f(t)dt - \frac{2}{\pi}x - 2$

[f(x) is periodic function with period π, therefore, $\int_{\pi}^{\pi+x} f(t)dt = \int_0^x f(t)dt$]

$$\Rightarrow p(x+\pi) = \int_0^{\pi} |\sin x|\, dx + g(x) - \frac{2}{\pi}x - 2$$

$$= 2 + g(x) - \frac{2}{\pi}x - 2$$

$p(x+\pi) = p(x)$ for all x

89. Let A be the set of vectors $a = (a_1, a_2, a_3)$ satisfying $\left(\sum_{i=1}^{3} \frac{a_i}{2^i}\right)^2 = \sum_{i=1}^{3} \frac{a_i^2}{2^i}$. Then

(A) A is empty
(B) A contains exactly one element
(C) A has 6 elements
(D) A has infinitely many elements

Ans. **[B]**

Sol. Given $\left(\sum_{i=1}^{3} \frac{a_i}{2^i}\right)^2 = \sum_{i=1}^{3} \frac{a_i^2}{2^i}$

$$\left(\frac{a_1}{2} + \frac{a_2}{4} + \frac{a_3}{8}\right)^2 = \frac{a_1^2}{2} + \frac{a_2^2}{4} + \frac{a_3^2}{8}$$

Simplifying , we get $\frac{a_1^2}{4} + \frac{a_2^2}{16} + \frac{a_3^2}{64} + \frac{a_1 a_2}{4} + \frac{a_1 a_3}{8} + \frac{a_2 a_3}{16} = \frac{a_1^2}{2} + \frac{a_2^2}{4} + \frac{a_3^2}{8}$

$16a_1^2 + 12a_2^2 + 7a_3^2 = 16a_1a_2 + 8a_1a_3 + 4a_2a_3$

$(8a_1^2 + 8a_2^2 - 16a_1a_2) + (8a_1^2 + 2a_3^2 - 8a_1a_3)$

$+ (4a_2^2 + a_3^2 - 4a_2a_3) + 4a_3^2 = 0$

$8(a_1 - a_2)^2 + 2(2a_1 - a_3)^2 + (2a_2 - a_3)^2 + 4a_3^2 = 0$

$$\Rightarrow \left.\begin{aligned} a_1 - a_2 &= 0 \\ 2a_1 - a_3 &= 0 \\ 2a_2 - a_3 &= 0 \\ a_3 &= 0 \end{aligned}\right\} \Rightarrow a_1 = a_2 = a_3 = 0$$

$\Rightarrow$ A contains exactly one element

90. Let $f : [0, 1] \to [0, 1]$ be a continuous function such that $x^2 + (f(x))^2 \le 1$ for all $x \in [0, 1]$ and $\int_0^1 f(x)dx = \frac{\pi}{4}$.

Then $\int_{\frac{1}{2}}^{\frac{1}{\sqrt{2}}} \frac{f(x)}{1 -} dx$ equals

(A) $\frac{\pi}{12}$
(B) $\frac{\pi}{15}$
(C) $\frac{\sqrt{2}-1}{2}\pi$
(D) $\frac{\pi}{10}$

Ans. **[A]**

Sol. $f(x) \le \sqrt{1 - x^2}$

$$\int_0^1 f(x)\,dx \le \int_0^1 \sqrt{1 - x^2}\,dx$$

$$\int_0^1 f(x)\,dx \le \left[\frac{x}{2}\sqrt{1 - x^2} + \frac{1}{2}\sin^{-1}(x)\right]_0^1$$

$$\Rightarrow \int_0^1 f(x)\,dx \le \frac{\pi}{4}$$

$$\Rightarrow f(x) = \sqrt{1 - x^2}$$

$$\Rightarrow \int \frac{f(x)}{1 - x^2}dx = \int_{1/2}^{1/\sqrt{2}} \frac{dx}{\sqrt{1 - x^2}}$$

$$= [\sin^{-1} x]_{1/2}^{1/\sqrt{2}} = \frac{\pi}{4} - \frac{\pi}{6} = \frac{\pi}{12}$$

PHYSICS

91. A metal rod of cross-sectional area 10^{-4} m^2 is hanging in a chamber kept at 20 °C with a weight attached to its free end. The coefficient of thermal expansion of the rod is 2.5×10^{-6} K^{-1} and its Young's modulus is 4×10^{12} N/m^2. When the temperature of the chamber is lowered to T then a weight of 5000 N needs to be attached to the rod so that its length is unchanged. Then T is

(A) 15 °C (B) 12 °C (C) 5 °C (D) 0 °C

Ans. **[A]**

Sol. $\frac{\Delta\ell}{\ell} = \alpha\Delta\theta$

$y = \frac{F\ell}{A\,\Delta\ell}$

Put the values

$\Delta\theta = 20 - T = 15$ºC

92. A short solenoid (length l and radius r, with n turns per unit length) lies well inside and on the axis of a very long, coaxial solenoid (length L, radius R and N turns per unit length, with R > r). Current I flows in the short solenoid. Choose the correct statement.

(A) There is uniform magnetic field $\mu_0 nI$ in the long solenoid.

(B) Mutual inductance of the solenoids is $\pi\mu_0 r^2 nNl$.

(C) Flux through outer solenoid due to current I in the inner solenoid is proportional to the ratio R/r.

(D) Mutual inductance of the solenoids is $\pi\mu_0\, rRnNlL(rR)^{1/2}$.

Ans. **[B]**

Sol. $M = \frac{\phi_{sec}}{i_{primary}}$

Smaller coil will act as secondary coil

$\phi = NaB$

$M = \frac{n\ell(\pi r^2)\mu_0 Ni}{i} = \mu_0 Nn\pi r^2\ell$

93. Consider the wall of a dam to be straight with height H and length L. It holds a lake of water of height h(h < H) on one side. Let the density of water be ρ_w. Denote the torque about the axis along the bottom length of the wall by τ_1. Denote also a similar torque due to the water up to height h/2 and wall length L/2 by τ_2. Then τ_1/τ_2 (ignore atmospheric pressure) is

(A) 2 (B) 4 (C) 8 (D) 16

Ans. [D]

Sol. $\tau = \rho g L\left[\frac{y^2h}{2} - \frac{y^3}{3}\right]_0^h$

$\tau_1 = \rho g L_1\left[\frac{h^3}{2} - \frac{h^3}{3}\right]$

similarly $\tau_2 = \rho g L_2\left[\frac{\frac{h^3}{2}}{6}\right]$

$\frac{\tau_1}{\tau_2} = 16$

94. Two containers C1 and C2 of volumes V and 4V respectively hold the same ideal gas and are connected by a thin horizontal tube of negligible volume with a valve which is initially closed. The initial pressures of the gas in C1 and C2 are P and 5P, respectively. Heat baths are employed to maintain the temperatures in the containers at 300 K and 400 K respectively. The valve is now opened. Select the correct statement:

(A) The gas will flow from the hot container to the cold one and the process is reversible.

(B) The gas will flow from one container to the other till the number of moles in two containers are equal.

(C) A long time after the valve is opened, the pressure in both the containers will be 3P.

(D) A long time after the valve is opened, number of moles of gas in the hot container will be thrice that of the cold one.

Ans. [D]

Sol. $\because P_1 = P_2$

$\frac{nRT}{V} = \text{Same}$

which gives $\frac{n_1}{n_2} = \frac{1}{3}$

95. Four electrons, each of mass m_e are in a one dimensional box of size L. Assume that the electrons are non-interacting, obey the Pauli exclusion principle and are described by standing de Broglie waves confined within the box. Define $\alpha = h^2/8m_eL^2$ and U_0 to be the ground state energy. Then

(A) the energy of the highest occupied state is 16α

(B) $U_0 = 30\alpha$

(C) the total energy of the first excited state is $U_0 + 9\alpha$

(D) The total energy of the second excited state is $U_0 + 8\alpha$

Ans. **[D]**

Sol. $\frac{n\lambda}{2} = L$

$\therefore P = \frac{h}{\lambda}$

$P = \frac{h\,n}{2L}$

$E = \frac{P^2}{2m} = n^2\alpha$

$E_1 = \alpha, E_2 = 4\alpha, E_3 = 9\alpha$

$E_3 = E_1 + 8\alpha$

96. A rope of length L and uniform linear density is hanging from the ceiling. A transverse wave pulse, generated close to the free end of the rope, travels upwards through the rope. Select the correct option.

(A) The speed of the pulse decreases as it moves up

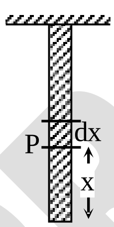
(B) The time taken by the pulse to travel the length of the rope is proportional to $\sqrt{L}$

(C) The tension will be constant along the length of the rope

(D) The speed of the pulse will be constant along the length of the rope.

Ans. **[B]**

Sol.



Tension at point P

$T = \frac{mg}{L}x = \mu g x$

$\because \mu = \sqrt{\frac{T}{\mu}} = \frac{dx}{dt} = \sqrt{gx}$

$\therefore \frac{dx}{\sqrt{x}} = \sqrt{g}\,dt$

By integrating

$t = \frac{2}{\sqrt{g}}(\sqrt{L})$

97. A circuit consists of a coil with inductance L and an uncharged capacitor of capacitance C. The coil is in a constant uniform magnetic field such that the flux through the coil is Φ . At time t = 0, the magnetic field is abruptly switched off. Let $\omega_0 = 1/\sqrt{LC}$ and ignore the resistance of the circuit. Then,

(A) current in the circuit is $I(t) = (\Phi/L)\cos\omega_0 t$

(B) magnitude of the charge on the capacitor is $|Q(t)| = 2C\omega_0|\sin\omega_0 t|$

(C) initial current in the circuit is infinite

(D) initial charge on the capacitor is $C\omega_0\Phi$

Ans. **[A]**

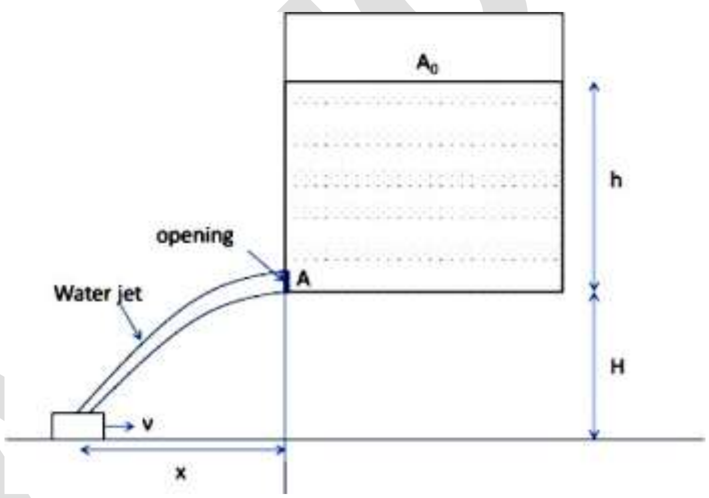
Sol. $\frac{q}{C} = -L\frac{di}{dt}$

$Li_0 = \Phi \qquad i_0 = \frac{\Phi}{L}$

$\therefore \frac{d^2q}{dt^2} = \frac{-q}{LC}$

Hence $i = \frac{\Phi}{L}\cos\omega_0 t$

98. Consider the configuration of a stationary water tank of cross section area A_0, and a small bucket as shown in figure below :



What should be the speed, ν, of the bucket so that the water leaking out of a hole of cross-section area A (as shown) from the water tank does not fall outside the bucket? Take h = 5 m, H = 5 m, g = 10 m/s^2, A = 5 cm^2 and A_0 = 500 cm^2.

(A) 1 m/s (B) 0.5 m/s (C) 0.1 m/s (D) 0.05 m/s

Ans. [C]

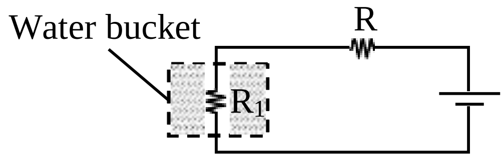
Sol. $x = \nu t = \sqrt{4yH}$

$\frac{dx}{dt} = \nu = \sqrt{4\frac{H}{y}} \times \frac{1}{2} \times \frac{dy}{dt}$

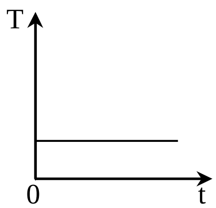
$\frac{dy}{dt} = \sqrt{2gy} \times \left(\frac{A}{A_0}\right)$

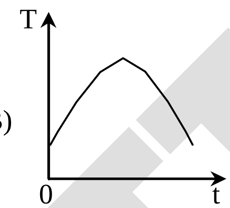
$\nu = \sqrt{\frac{H}{y}} \times \sqrt{2gy} \times \frac{A}{A_0} = 0.1$ m/sec

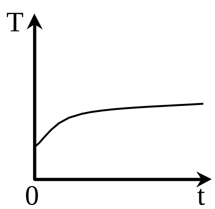
99. The circuit below is used to heat water kept in a bucket

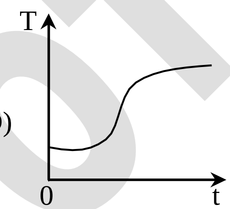


Assuming heat loss only by Newton's law of cooling, the variation in the temperature of the water in the bucket as a function of time is depicted by :

(A)


(B)


(C)


(D)


Ans. **[C]**

Sol. $\frac{dQ}{dt} = ms\frac{d\theta}{dt}$

$\frac{dQ}{dt} \propto \theta - \theta_0$

100. A bubble of radius R in water of density ρ is expanding uniformly at speed ν. Given that water is incompressible, the kinetic energy of water being pushed is

(A) Zero (B) $2\pi\rho R^3\nu^2$ (C) $2\pi\rho R^3\nu^2/3$ (D) $4\pi\rho R^3\nu^2/3$

Ans. **[B]**

Sol. As bubble increases its volume, due to surface tension work is to be done, velocity flux will be same.

$\therefore\ dk = \frac{1}{2}d(m\nu^2)$

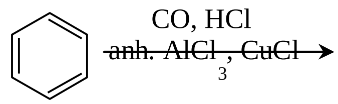
$= \frac{\rho}{2}\left[4\pi x^2 dx \frac{R^4\nu^2}{x^4}\right]$

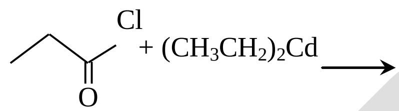
By integrating x = R to ∞

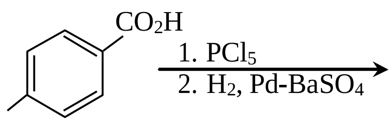
$\Delta k = 2\pi\rho R^3\nu^2$

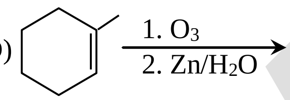
CHEMISTRY

101. The product of which of the following reaction forms a reddish brown precipitate when subjected to Fehling's test?

(A) 

(B) 

(C) 

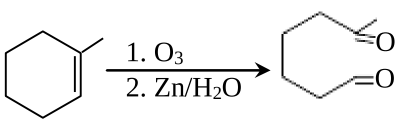
(D) 

Ans. **[D]**

Sol. Fehling test - : Fehling solution A + Fehling solution B
(Cu^{2+}) OH^-

Felling reagent reacts with aldehyde and a reddish brown precipitate is obtained

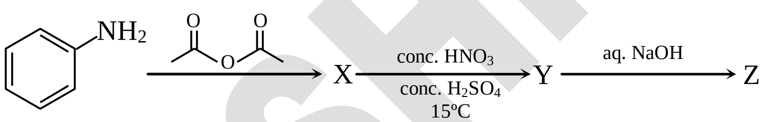
Reaction : - $R-CHO + 2Cu^{2+} + 5\,\overset{\ominus}{O}H \longrightarrow RCO\overset{\ominus}{O} + Cu_2O + 3H_2O$ Red-brown ppt.

Option (D) 

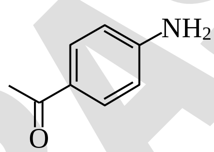
$\Rightarrow CH_3-\underset{\overset{\|}{O}}{C}-CH_2-CH_2-CH-CH_2-\underset{\overset{\|}{O}}{C}-H$

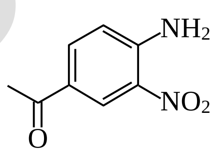
aldehyde

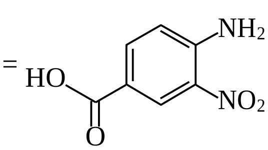
102. The major products X, Y and Z in the following sequence of transformations

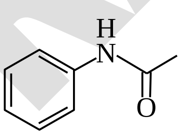


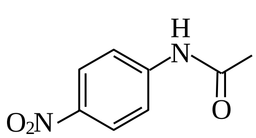
are

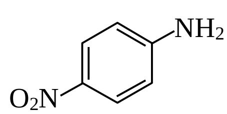
(A) X = 

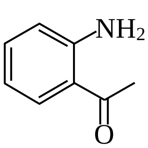
Y = 

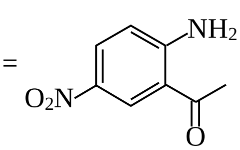
Z = 

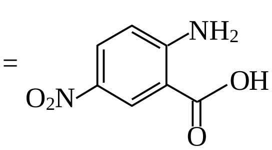
(B) X = 

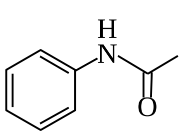
Y = 

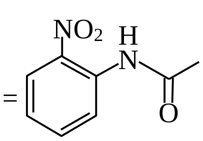
Z = 

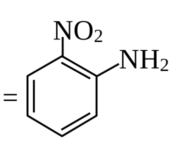
(C) X = 

Y = 

Z = 

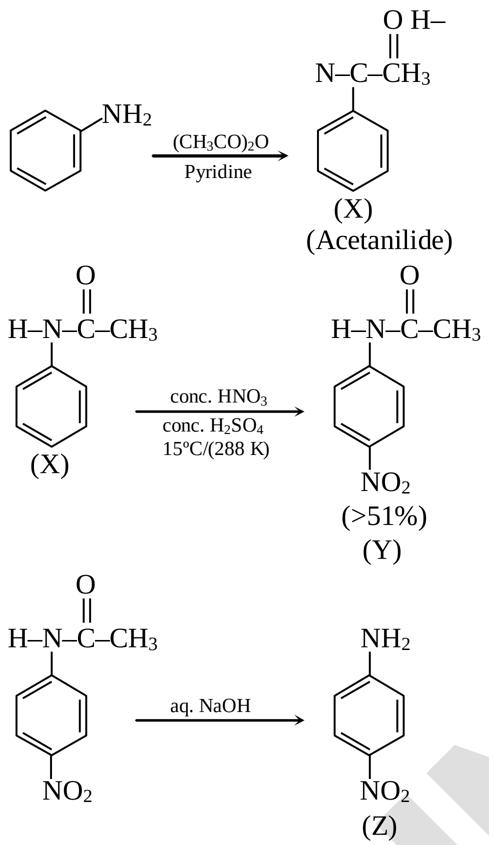
(D) X = 

Y = 

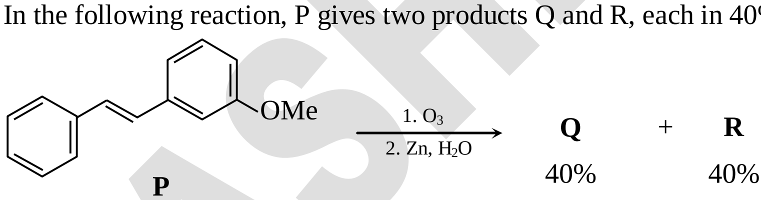
Z = 

Ans. **[B]**

Sol.



103. In the following reaction, P gives two products Q and R, each in 40% yield.



P

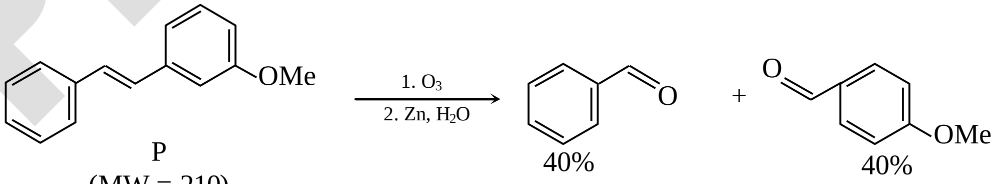
(MW = 210)

If the reaction is carried out with 420 mg of **P**, the reaction yields 108.8 mg of **Q**. The amount of **R** produced in the reaction is closets to

(A) 97.6 mg (B) 108.8 mg (C) 84.8 mg (D) 121.6 mg

Ans. **[C]**

Sol.



P

(MW = 210)

420 mg

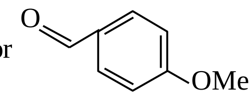
$\frac{420}{210} = 2$ milimoles

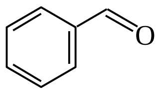
$2\times\frac{40}{100}$ $\qquad$ $2\times\frac{40}{100}$

FORMED = $\frac{4}{5}$ mole $\qquad$ $\frac{4}{5}$ mole

MOLES

Molecular weight for 108.8 mg = $\frac{4}{5} \times 108.8$ mg = 136

136 g/mol is for  $\Rightarrow$ Q

Then R $\Rightarrow$  $\Rightarrow C_7H_6O \Rightarrow$ Mol. wt = 106

moles of R = $\frac{4}{5} \Rightarrow$ mass of R = $\frac{4}{5} \times 106 = 84.8$ mg

$\boxed{\text{mass of R} = 84.8 \text{ mg}}$

104. Solubility products of CuI and Ag_2CrO_4 have almost the same value ($\sim 4 \times 10^{-12}$). The ratio of solubilities of the two salts (CuI : Ag_2CrO_4) is closest to

(A) 0.01 (B) 0.02 (C) 0.03 (D) 0.10

Ans. **[B]**

Sol. K_{sp} (CuI) = 4×10^{-12}

K_{sp} (Ag_2CrO_4) = 4×10^{-12}

$$\underset{}{CuI_{(s)}} \longrightarrow \underset{\text{"}S_1\text{"}}{Cu^{\oplus}_{(aq)}} + \underset{\text{"}S_1\text{"}}{I^{\ominus}_{(aq.)}}$$

$K_{sp_1} = S_1 \times S_1 \Rightarrow 4 \times 10^{-12} = S_1^2$

$\boxed{S_1(\text{CuI}) = 2 \times 10^{-6} \text{ mol/L}}$

$K_{sp}(Ag_2CrO_4) = 4 \times 10^{-12}$

$$Ag_2CrO_4 \longrightarrow \underset{2\text{'}S_2\text{'}}{2Ag^+} + \underset{S_2}{CrO_4^{2-}}$$

$K_{sp_2} = (2S_2)^2(S_2)$

$4 \times 10^{-12} = 4S_2^3$

$S_2^3 = 10^{-12} \Rightarrow \boxed{S_2 = 10^{-4} \text{ mol/L}}$

$$\frac{\text{SOLUBILITY of CuI}}{\text{SOLUBILITY of } Ag_2CrO_4} = \frac{S_1}{S_2} = \frac{2 \times 10^{-6}}{10^{-4}} = 2 \times 10^{-2}$$

$$\boxed{\frac{S_1}{S_2} = 0.02}$$

105. Given that the molar combustion enthalpy of benzene, cyclohexane, and hydrogen are x, y, and z, respectively, the molar enthalpy of hydrogenation of benzene to cyclohexane is

(A) $x - y + z$ (B) $x - y + 3z$ (C) $y - x + z$ (D) $y - x + 3z$

Ans. **[B]**

Sol. $C_6H_6 + \frac{15}{2}O_2 \longrightarrow 6CO_2 + 3H_2O$; x …(1)

$C_6H_{12} + 9O_2 \longrightarrow 6CO_2 + 6H_2O$; y … (2)

$H_2 + \frac{1}{2}O_2 \longrightarrow H_2O$; z … (3)

$C_6H_6 + 3H_2 \longrightarrow C_6H_{12}$; ? … (4)

(1) + 3 × (3) – (2) : to get reaction (4) :

$C_6H_6 + 3H_2 \longrightarrow C_6H_{12} \Rightarrow x + 3z - y$

$= x - y + 3z$

[Molar enthalpy of hydrogenation of benzene to cycle hexane = x – y + 3z]

106. Among the following, the pair of paramagnetic complexes is

(A) $K_3[Fe(CN)_6]$ and $K_3[CoF_6]$ (B) $K_3[Fe(CN)_6]$ and $[Co(NH_3)_6]Cl_3$

(C) $K_4[Fe(CN)_6]$ and $K_3[CoF_6]$ (D) $K_4[Fe(CN)_6]$ and $[Co(NH_3)_6]Cl_3$

Ans. **[A]**

Sol. Paramagnetic complexes

$\Downarrow$

Unpaired electron present

(A) $K_3[Fe(CN)_6]$

$\downarrow$

Fe^{+3}

$Fe(26) \Rightarrow [Ar]\ 3d^6 4s^2$

$Fe^{3+} \Rightarrow [Ar]\ 3d^5$

$CN^{\ominus}$ strong field ligand

1 1 1 1 1

→ Pairing possible

⥮	⥮			

$\boxed{\text{No.of unpaired } e^- = 1}$

1

$\Downarrow$

paramagnetic

$K_3[CoF_6]$

$\downarrow$

Co^{3+}

$Co(27) \Rightarrow [Ar]\ 3d^7\ 4s^2$

$Co^{3+} \Rightarrow [Ar]\ 3d^6$

⥮				

$F^{\ominus}$ weak field ligand

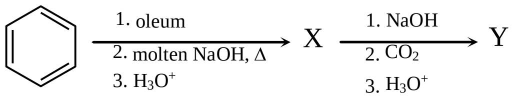
No pairing

$\boxed{\text{No.of unpaired } e^- = 4}$

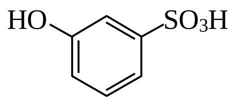
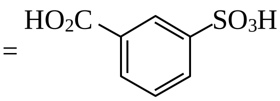
$\Downarrow$

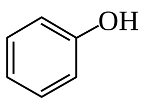
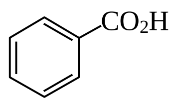
paramagnetic

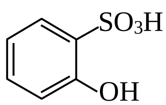
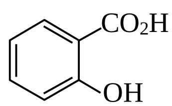
107. The major products X and Y in the following sequence of transformations

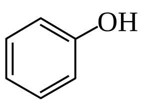
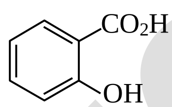


are

(A) X =  Y = 

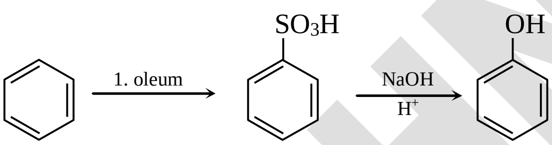
(B) X =  Y = 

(C) X =  Y = 

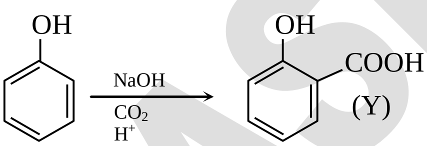
(D) X =  Y = 

Ans. **[D]**

Sol.

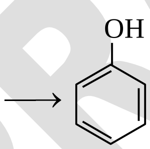


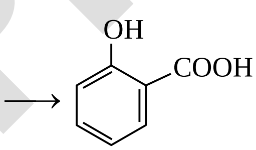
(X)



(X) (Y)

Salicylic Acid

X 

Y 

108. 3.0 g of oxalic acid $[(CO_2H)_2.2H_2O]$ is dissolved in a solvent to prepare a 250 mL solutions. The density of the solution is 1.9 g/mL. The molality and normality of the solution, respectively, are closest to

(A) 0.10 and 0.38 (B) 0.10 and 0.19 (C) 0.05 and 0.19 (D) 0.05 and 0.09

Ans. **[C]**

Sol. Given - m = 3.0 g

$(COOH)_2.2H_2O$ M = 90 + 36 = 126

$\rho = 1.9$ g/ml

V = 250 ml $\rho = \frac{M}{V} \Rightarrow M = \rho \times V$

$$m = \frac{n_{solute}}{m_{solvent}(Kg)} = \frac{m/M}{\rho / V}$$

$$= \frac{3/126}{1.9 \times 250 \times 10^{-3}} = \frac{10^3}{42 \times 19 \times 25}$$

$$\boxed{m = 0.05}$$

n_f of oxalic acid = 2

$N = M \times n_f$

$$N = \frac{n}{V(L)} \times n_f$$

$$= \frac{3/126}{250 \times 10^3} \times 2$$

$$= \frac{40}{42} \times 2$$

$$\boxed{N = 0.19}$$

109. In a titration experiment, 10 mL of an $FeCl_2$ solution consumed 25 mL of a standard $K_2Cr_2O_7$ solution to reach the equivalent point. The standard $K_2Cr_2O_7$ solution is prepared by dissolving 1.225 g of $K_2Cr_2O_7$ in 250 mL water. The concentration of the $FeCl_2$ solution is closest to

[Given : molecular weight of $K_2Cr_2O_7$ = 294 g mol^{-1}]

(A) 0.25 N (B) 0.50 N (C) 0.10 N (D) 0.04 N

Ans. **[A]**

Sol.
$$FeCl_2 + K_2\underbrace{\overset{+6}{Cr_2}O_7 \longrightarrow Fe^{+3} + Cr^{+3}}_{n_f = 6}$$

no. of eq. of $FeCl_2$ = No. of eq. of $K_2Cr_2O_7$

$N_1V_1 = N_2V_2$

Given : - V_1 = 10 ml

V_2 = 25 ml

N_1 = ?

m = 1.225 g M = 294 g/mol

$\Rightarrow N_1 \times 10 = M \times n_f \times 25$

$$N_1 \times 10 = \frac{n}{V(L)} \times n_f \times 25$$

$$N_1 \times 10 = \frac{1.225 / 294}{250 \times 10^{-3}} \times 25$$

$$N_1 = \frac{1.225 \times 25}{294 \times 250 \times 10 \times 10^{-3}} = \frac{1.225}{10 \times 10 \times 10^{-3} \times 294}$$

$$N_1 = \frac{12.25}{294} = 0.25$$

$$\boxed{N_1 = 0.25N}$$

110. Atoms of an element Z form hexagonal closed pack (hcp) lattice and atoms of element X occupy all the tetrahedral voids. The formula of the compound is

(A) XZ (B) XZ_2 (C) X_2Z (D) X_4Z_3

Ans. **[C]**

Sol. In hcp Z = 6

no. of tetrahydral voids = 2 × Z

= 2 × 6 = 12

no. of atoms of X = 12

Z is in hcp

no. of Z in 1 unit cell is = 6

X : Z

12 : 6 ⇒ $\boxed{X_2Z}$

BIOLOGY

111. In a population, N_{AA} and N_{aa} are the numbers of homozygous individuals of allele 'A' and 'a', respectively, and N_{Aa} is the number of heterozygous individuals. Which one of the following options is the allele frequency of 'A' and 'a' in a population with N_{AA} = 90, N_{Aa} = 40 and N_{aa} = 70?

(A) A = 0.55 and a = 0.45 (B) A = 0.40 and a = 0.60

(C) A = 0.35 and a = 0.65 (D) A = 0.25 and a = 0.75

Ans. **[A]**

Sol.

112. A newly discovered organism possesses a genetic material with a new base composition consisting of the sugar and phosphate backbone as found in existing natural DNA. The five novel bases in this genetic material – namely, P, Q, R, S, T – are heterocyclic structures with 1, 1, 2, 2, and 3 rings, respectively. Assuming the new DNA forms a double helix of uniform width, which one of the following would be the most appropriate base pairing.

(A) P with Q; R with T; S with T (B) P with T; R with S; Q with T

(C) P with S; Q with R; S with T (D) P with Q; R with S; S with T

Ans. **[B]**

Sol. No. of chains in basses.

P = 1

Q = 1

R = 2

S = 2

T = 3

P T	R S	Q T
1 +3	2 + 2	1 + 3
4	4	4

so when the PT, RS, QT bond then the width will be same.

113. Amino acid analysis of two globular protein samples yielded identical composition per mole. Which one of the following characteristics is necessarily identical for the two proteins ?

(A) Disulphide bonds (B) Primary structure

(C) Molecular mass (D) Three-dimensional structure

Ans. **[C]**

Sol. If the composition per mole of 2 proteins are identical then their molecular mass must be identical.

114. Which of the following conversions in glycolysis is an example of substrate level phosphorylation ?

(A) Glyceraldehyde-3-phosphate to 1,3-bisphosphoglycerate

(B) 1,3-bisphosphoglycerate to 3-phosphoglycerate

(C) Fructose 6-phosphate to fructose-1,6-bisphosphate

(D) Glucose-6-phosphate to fructose-6-phosphate

Ans. **[B]**

Sol. 1,3-bisphosphoglycerate to 3-phosphoglycerate

Substrate level phosphorylation refers to the formation of ATP from ADP and a phosphorylated intermediate, rather then from ADP and ip (inorganic phosphate).

Substrate level phosphorylation take place in the two steps of glycolysis.

(i) 1,3-bisphosphoglycerate to 3-phosphoglycerate

(ii) Phosphoenol pyruvate (PEP) to Pyruvic acid.

115. A plant heterozygous for height and flower colour (TtRr) are selfed and 1600 of the resulting seeds are planted. If the distance between the loci controlling height and flower colour is 1 centimorgan, then how many offspring are expected to be short with white flower (ttrr) ?

(A) 1 (B) 10 (C) 100 (D) 400

Ans. **[A]**

Sol.

Parents TTRR × ttrr

↓ ↓

Gametes TR tr

F_1 - offspring TtRr selfing

Possible Gametes

TR Tr tR tr

♀ \ ♂	TR	Tr	tR	tr
TR	TTRR	TTRr	TtRR	TtRr
Tr	TTRr	TTrr	TtRr	Ttrr
tR	TtRR	TrRr	ttRR	ttRr
tr	TrRr	Ttrr	ttRr	ttrr

TTRR : TTrr : ttRR : ttrr

900 : 300 : 300 : 100 = 1600

900 : 300 : 300 : 100 $\Rightarrow$ 9 : 3 : 3 : 1

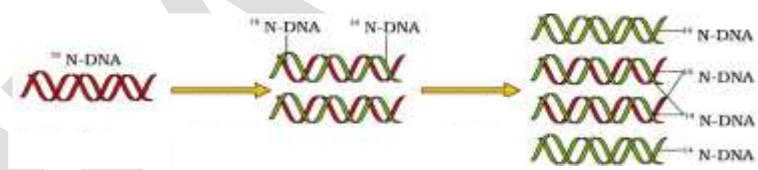
ttrr (white flower) = 1

116. Which one of the following will be the ratio of heavy, intermediate and light bands in Meselson and Stahl's experiment after two generations if DNA replication were conservative ?

(A) 0:2:2 (B) 1:0:3 (C) 2:2:0 (D) 2:0:2

Ans. **[B]**

Sol.



Radio heavy : intermediate : light

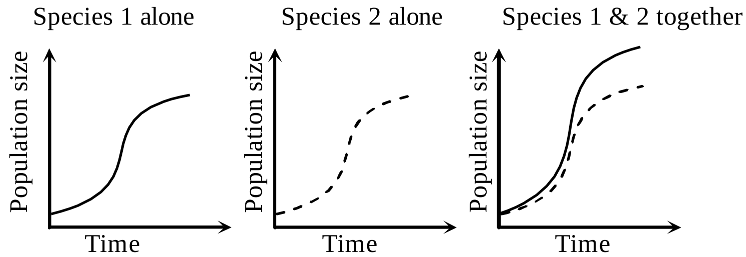
1 : 0 : 3

N^{15} : Radioactive heavy nitrogen

N^{14} : - Light weight Nitrogen

$N^{15}N^{14} \rightarrow$ Intermediate

117. Given the graphs below, the interaction between species 1 and 2 can be classified as



(A) amensalism (B) commensalism (C) mutualism (D) competition

Ans. **[B]**

Sol. When species 1 & 2 together, species 1 is getting benifit while species 2 having neither benifited nor in loss. This type is commensalism.

118. The additional nuclear ploidy levels found in a diploid angiosperm species in full bloom compared to its vegetative stage are

(A) 1N & 2N (B) 2N & 3N (C) 3N & 4N (D) 1N & 3N

Ans. **[D]**

Sol. Since the species is diploid so its ploidy will be 2N besides this other ploidy found are 1N (gametes) & 3N(endosperm)

119. The bill sizes in a bird species of seedcrackers from West Africa shows a bimodal distribution. Their most abundant food sources are two types of marsh plants that produce hard and soft seeds, consumed preferentially by the large and small billed birds respectively. This bimodal distribution of bill sizes is a likely consequence of

(A) directional selection (B) stabilising selection (C) disruptive selection (D) sexual selection

Ans. [C]

Sol. When food is abundant in environment and both. Small billed and large billed are adaptive for this type of situation.

So this type of consequence will be showing disruptive selection.

120. The containers X and Y have 1 litre of pure water and 1 litre of pure water and 1 litre of 0.1 M sugar solution, respectively. Which one of the following statements would be CORRECT regarding their water potential (Ψ) and osmotic potential (Ψ_S) ?

(A) Both Ψ and Ψ_S are zero in X (B) Both Ψ and Ψ_S are zero in Y

(C) Ψ in X is zero and Ψ_S in Ψ is negative (D) Ψ in X is negative and Ψ_S in Y is zero

Ans. [C]

Sol. Container X contains pure water

So Ψ = O (maximum)

contains Y contains 0.1 sugar solution.

So Ψ_s = –ve

$\Psi = \Psi_s + \Psi_p$

Any solute will always lower the water potential of pure water.